Geodätisch-geophysikalische Arbeiten in der Schweiz

(Fortsetzung der Publikationsreihe
«Astronomisch-geodätische Arbeiten in der Schweiz»)

herausgegeben von der

Schweizerischen Geodätischen Kommission
(Organ der Schweizerischen Akademie der Naturwissenschaften)

Vierundvierzigster Band

Rapid Differential Positioning with the Global Positioning System (GPS)

Erwin Frei

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VORWORT

Nachdem die Schweizerische Geodätische Kommission (SGK) bereits Mitte der 80-er Jahre in einer Spezial-Arbeitsgruppe des Europarates SATRAPE (Satellite Radio Positioning in Europe) ihre GPS-Aktivitäten aufgenommen und eine Reihe von nationalen Projekten angeregt hatte (Testnetz Turtmann, Nationales Forschungsprogramm Geologische Tiefenstruktur der Schweiz), freuen wir uns, nun auch auf dem Gebiet der Detail- und Ingenieurvermessung einen wichtigen Meilenstein vorstellen zu dürfen.

Die vorliegende Dissertation von Herrn Dr. Erwin Frei entstand im Rahmen des Projektes Nr. 1584 der Schweizerischen Kommission zur Förderung der Wissenschaftlichen Forschung (KWF). Primäres Ziel war es, die Vermessung in kleinen Gebieten (bis maximal etwa 10 km x 10 km) mit dem amerikanischen Global Positioning System (GPS) eingehend zu untersuchen und zu optimieren. Dabei war von Anfang an klar, dass dem Bestimmen der Phasenmehrdeutigkeiten ("ambiguity resolution process") eine zentrale Bedeutung zukommt: Sind diese Werte bekannt, genügen sehr kurze Messreihen (1- 5 Minuten) zur Erzielung relativer Punktgenauigkeiten von einem Zentimeter. Sind sie unbekannt, liefert die gleiche Messreihe nur eine relative Genauigkeit von einigen Dezimetern. Kern der Arbeit ist daher die mathematische Analyse dieses Prozesses zur Bestimmung der Phasenmehrdeutigkeiten.

Ursprünglich war an Messreihen von ungefähr 15 - 30 Minuten gedacht. Herrn Erwin Frei gelang es jedoch zu zeigen, dass bei Verwendung von Zweifrequenzen-Geräten schon Messreihen von wenigen Minuten ausreichen. Dies konnte a priori kaum erwartet werden. Das wichtigste Resultat der vorliegenden Arbeit ist ein neues Verfahren, von Herrn Frei "Rapid Static Positioning" genannt, welches in naher Zukunft die Detail- und Ingenieur- Vermessung wesentlich mitgestalten, wenn nicht revolutionieren wird.

Die Schweizerische Geodätische Kommission dankt Herrn Dr. Erwin Frei für seinen wertvollen Beitrag zur Geodäsie und Landesvermessung der Schweiz. Von der Schweizerischen Akademie der Naturwissenschaften (SANW) wurden die Druckkosten übernommen, wofür die SGK ihren Dank ausspricht.

Im Namen der Schweizerischen Geodätischen Kommission:

Prof. Dr. G. Beutler
Vorsteher Astronomisches
Institut der Universität Bern

Direktor F. Jeanrichard
Vizepräsident der SGK

Prof. Dr. H.-G. Kahle
Präsident der SGK

PREFACE

Au milieu des années 80 déjà, la Commission géodésique suisse (CGS) a entrepris des activités dans le domaine GPS au sein du groupe de travail SATRAPE du Conseil de l'Europe (Satellite Radio Positioning in Europe) et proposé une série de projets nationaux (Réseau-test de Turtmann, programme national de recherche "Exploration du soubassement géologique de la Suisse"). Aujourd'hui, nous avons le plaisir de présenter une étape marquante dans les domaine des levés de détail et de la mensuration technique et industrielle.

La dissertation de Monsieur Erwin Frei, dr ès sc.techn., présentée ici, fait partie du projet no 1584 de la Commission pour l'encouragement de la recherche scientifique (CERS). Son objectif principal est de tester et d'optimiser une mensuration basée sur le système américain Global Positioning System (GPS) sur de petites surfaces (au maximum 10 km x 10 km). D'emblée, il s'avéra que la résolution des abiguïtés des phases ("ambiguity resolution process") avait une importance déterminante: si ces valeurs sont connues, de très courtes séries de mesures (1-5 minutes) suffisent dès lors pour obtenir une précision relative de l'ordre du centimètre. Si ces valeurs sont inconnues, la même série de mesures ne permet d'obtenir qu'une précision de l'ordre de quelques décimètres. L'objet principal de la recherche est donc l'analyse mathématique du procédé de la résolution des ambiguïtés des phases.

Au début, il était question de séries de mesures d'environ 15 à 30 minutes. Monsieur Erwin Frei a réussi à démontrer que l'utilisation de récepteurs à deux fréquences réduisait ces temps de mesures à quelques minutes, ce qui, à priori, n'était pas envisageable. Le résultat principal des travaux présentés ici est un nouveau procédé, dénommé "Rapid Static Positioning" par Monsieur Frei et qui, dans un avenir proche, aura des répercussions marquantes, sinon révolutionnaires, sur les levés de détail et la mensuration technique et industrielle.

La Commission géodésique suisse remercie Monsieur Erwin Frei, dr ès sc. techn., de sa précieuse contribution à la géodésie et à la mensuration nationale de la Suisse. Elle remercie également l'Académie suisse des sciences naturelles (ASSN) de la prise en charge des coûts d'impression.

Au nom de la Commission gédésique suisse:

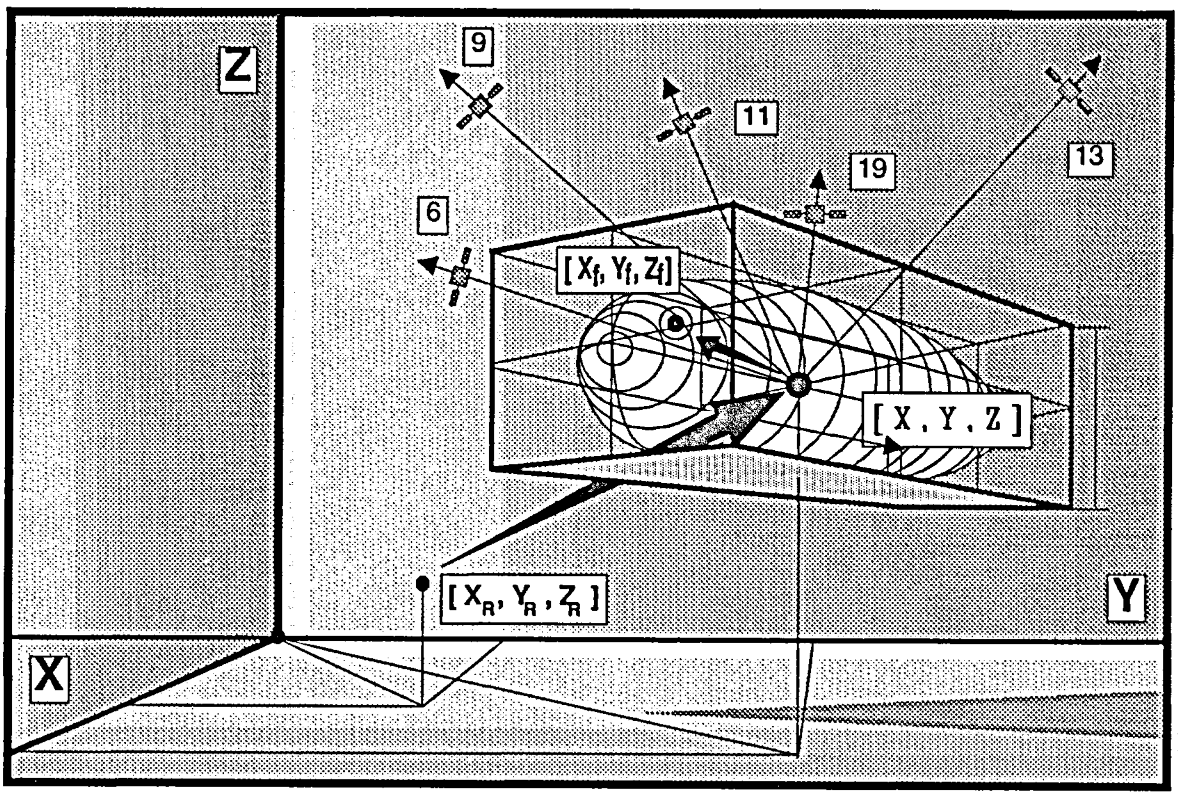
Prof. Dr. G. Beutler
Directeur de
l'Institut d'astronomie
de l'Université de Berne

F. Jeanrichard, directeur,
Vice-président de la CGS

Prof. Dr. H.-G. Kahle
Président de la CGS

Rapid Differential Positioning with the Global Positioning System (GPS)

Erwin Frei



Widnau und Bern

1990

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Lebenslauf

Part I : Introduction

Introduction

and

Summary

I : 1. The Global Positioning System (GPS)

Artificial earth satellites have been used for more than 20 years for precise geodetic positioning. The use of the so called TRANSIT satellite system, which has been available for civilian purposes since 1967, allows to determine point positions in the decimeter range with several hours worth of Doppler measurements. This satellite system has therefore been used mainly for navigation, in prospecting for natural resources and in establishing control for geodetic networks. This situation has changed with the introduction of the NAVSTAR Global Positioning System (NAVigation System Using Time And Ranging). The NAVSTAR GPS has been initiated and developed by the US Department of Defense (DoD). The initial intention was to use this satellite system mainly for navigation purposes within the US - military. Due to the tremendous accuracy potential of this system and the latest improvements in receiver technology there is a growing community which utilizes the GPS for a variety of civilian applications (navigation, geodetic positioning, etc.).

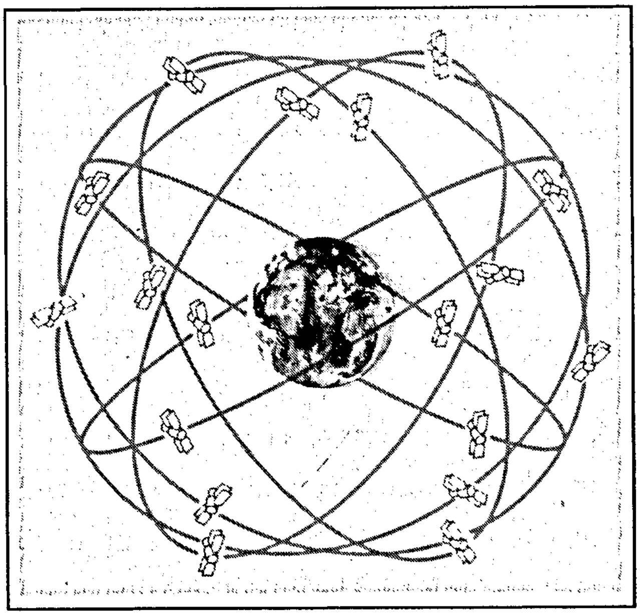

Figure 1.1 : The GPS satellite constellation in its ultimate stage of deployment

There is a similar satellite system to the NAVSTAR GPS being developed by the Union of Socialist Soviet Republic (U.S.S.R.). This system is known under the name GLONASS (GLObal NAvigation Satellite System). Apart from a few technical differences (e.g. all satellites transmit on different frequencies) the two systems are based on the same principles and ideas. The GLONASS is getting more and more attention because future receiver generations will be able to track satellites from both systems simultaneously. The satellite constellation of such a combined system would highly improve the positioning performance in terms of accuracy and also in terms of productivity (shorter site occupation times). Let us concentrate now on the NAVSTAR GPS for a brief description of the basics of such a satellite system.

After the full deployment the NAVSTAR GPS consists of 24 satellites (21 + 3 active spares) which are distributed in 6 orbital planes. Each plane has an inclination of 55 degrees relative to the equatorial plane (see Figure 1.1). All the satellites are at about 20,200 km above the earth. Every satellite completes a full revolution around the earth in approximately 11 hours and 58 minutes and therefore passes its starting position at the end of a sidereal day.

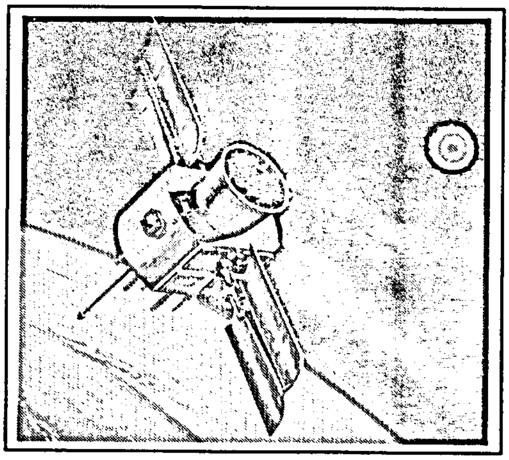

Figure 1.2 : An artist's view of a GPS satellite

The current NAVSTAR GPS satellite constellation almost provides the so called two-dimensional coverage. This means that at every instant of time everywhere on the earth's surface at least three satellites can be observed. This allows to determine the geocentric position of a GPS antenna in real-time with an accuracy of 5 to 30 meters if the height of the receiver is assumed known. There are already nowadays locations where up to six satellites can be observed simultaneously. It is expected hat the NAVSTAR Global Positioning System will be fully operational in 1992 to 1993.

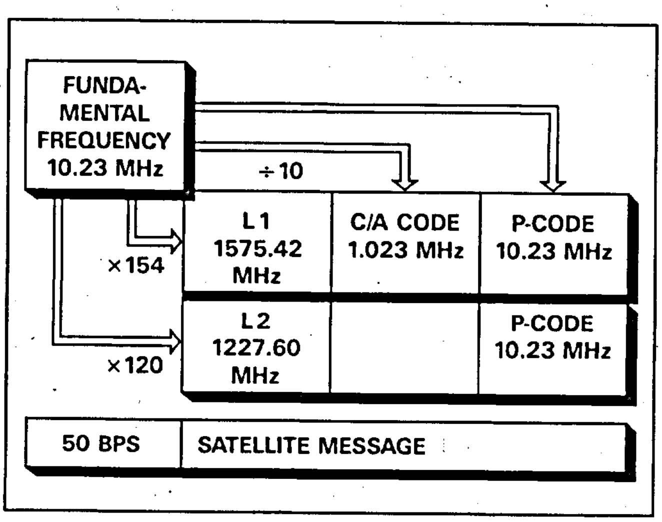

Figure 1.3 : Frequency and signal generation for a particular GPS satellite

Every satellite transmits continuously signals on two L - band carrier frequencies. The first carrier frequency L_1 is at 1575.42 MHz and the second carrier frequency L_2 is at 1227.60 MHz. The corresponding wavelengths are about 19 centimeters for L_1 and about 24 centimeters for the L_2 carrier frequency. Both carrier frequencies are modulated by so called "Pseudo-Random Noise Codes". The L1 carrier only is modulated by the "Coarse Acquisition Code" referred to as C/A - Code, whereas both carrier frequencies are modulated by the "Precise Code" referred to as P - Code. These two codes form the basis for the "Standard Positioning Service (SPS)" and the "Precise Positioning Service (PPS)". In addition, both signals contain the "Navigation Messages" which are transmitted with a data rate of 50 bits per second. These "Navigation Messages" contain information like e.g. ephemerides, satellite clock corrections, status and health information for each individual satellite and so on. Figure 1.3 shows a scheme for the frequency and signal generation for a particular satellite. After the full deployment of the system the access to the P - Code will be restricted to authorized users only. The stability of the carrier frequencies is guaranteed by the use of atomic clocks on board of each space vehicle.

Two different types of measurements can be taken when tracking GPS satellites : The "Pseudo - Range Measurements" and the "Carrier Beat Phase Measurements" (for a detailed introduction see [REMONDI, 1984]). The two types of measurements can be described in simplified terms as follows :

(i) "Pseudo - Range Measurements" : The distance from a specific satellite to a particular antenna can be determined if the propagation time of a signal is known (the range is equal the propagation time multiplied by the speed of light). The satellites transmit signals which are labeled with the exact time of transmission given in the GPS time system. The receivers measure the exact time of reception of such a signal relative to the receiver clock. Provided the receiver clock is fully synchronized to the GPS time system, then the time difference between the transmission time and the reception time is exactly the travel time of the signal. Due to the fact that the receiver clocks are usually not fully synchronized to the GPS time system the ranges determined following this procedure are wrong by the receiver synchronization error. These ranges are therefore referred to as "Pseudo-Ranges". Taking such "Pseudo-Range Measurements" simultaneously to four or more satellites allows to determine the geocentric position <u>and</u> the clock synchronization error in real-time. Depending on the availability of P - Code or C/A - Code signals the accuracy for these measurements is in the order of 3 to 10 meters or 15 to 30 meters.

(i) "Carrier Beat Phase Measurements" : The transmitted carriers can be reconstructed in the receivers if the pseudo - random codes (C/A - Code or P - Code) are known or if the receiver acquires the carriers by squaring - techniques. The phases of these incoming carriers (L_1 and/or L_2) are differenced with the phase of internally generated carriers. Such a difference in phase is referred to as "Carrier Beat Phase Measurement". Today's receiver technology allows to measure with an accuracy below the millimeter level. These quantities would represent very accurate range measurements if the so called "Initial Phase Ambiguities" could be determined. The term "Initial Phase Ambiguity" denotes the number of full cycles of the L_1 or L_2 carrier contained in the range from a particular satellite to the antenna for the epoch of the very first measurement. Once determined, these "Initial Phase Ambiguities" are valid as long as the receiver can keep lock to the satellites.

These two types of measurements are utilized for a variety of different positioning applications i.e. navigation, static differential positioning, kinematic differential positioning, high precision navigation, orbit determination, time transfer and so on. We will concentrate in the following sections on the aspects of differential positioning.

I : 2. Positioning with GPS

Five years ago, the main usage of the NAVSTAR Global Positioning System (GPS) for geodetic purposes was in the field of static differential positioning. Receivers were typically kept on individual sites for several hours or at least as long as a reasonable satellite constellation (three or more satellites) could be observed. The primary objective was accuracy. The required site occupation time and hence the productivity played a subordinate role. As a matter of consequence the main applications of these GPS based positioning techniques were to establish local, regional or even global networks which in turn served several geodetic purposes like e.g. geodynamic studies for densifications of VLBI- (Very Long Baseline Interferometry) and SLR- (Satellite Laser Ranging) networks or for local control etc. Thanks to the tremendous accuracy potential of static differential positioning techniques (a few millimeters for very short baselines to accuracies of 10^{-6} to 10^{-8} for distances up to 4000 kilometers), there are almost no restrictions as to where these techniques could be utilized for geodetic and surveying applications. At that time, this fact by itself was recognized as a revolutionary step in the field of positioning. With the proposal by B. Remondi in 1985 (REMONDI, 1985) to use the GPS receivers in a so called kinematic mode, the range of candidate applications of GPS based positioning was even further enlarged. The kinematic technique makes use of the fact that GPS receivers are capable to track the satellite-phases continuously even while the receivers are moving. Using this technique, point positioning can be performed much faster (seconds rather than hours) with subcentimeter accuracies. Based on the concepts of kinematic surveying several slightly modified techniques were proposed by different authors e.g. (Goad, Hatch, Wübbena, Blewitt). A few of them make use of phase measurements only, whereas others employ in addition P-code measurements either on the L_2 signal only or on both GPS signals (L_1 and L_2). The obvious advantage of positioning techniques based on the concepts of kinematic surveying is to be seen in the fact, that the surveying productivity is very high. This in turn makes this technique a real competitor to classical surveying methods (e.g. total stations). There are two different variants of the kinematic approach each focusing on different target applications. Firstly, the stop and go kinematic technique, where the movement of the receiver from one survey site to the next is of no interest. Only the survey sites are positioned. Therefore this technique fits nicely the requirements for detail survey applications. The second approach is the true kinematic case where the trajectory of the antenna is of primary interest. This technique is employed e.g. for high precision navigation, for determinations of camera positions for photogrammetry or for hydrographic surveys etc.

Recent advances in GPS positioning techniques are the proposals by B. Remondi [REMONDI, 1990], G. Beutler [BEUTLER et al., 1989] and V. Ashkenazi [ASHKENAZI et al., 1989] dealing with a repetitive occupation within one to two hours of one and the same site for very short intervals. The main objective of these repetitive site occupations is the strengthening of the geometrical content of the point determination by superimposing different satellite constellations. This results in much better conditioned normal equation systems than could be obtained by a single occupation with a short observation period. Literally, these reoccupation approaches, or as B. Remondi calls them [REMONDI, 1990] pseudo-kinematic approaches, are simply special cases of static positioning, because it is not required to keep lock to the satellites while travelling between different sites. Every site occupation can be treated independently as far as ambiguity parameters are concerned.

All the positioning techniques outlined above have one key issue in common, namely the resolution of initial phase ambiguities. The ability to resolve initial phase ambiguities represents the key to high precision positioning with GPS. Results on the sub-centimeter level are achievable if ambiguities can be fixed to integer values [W. Gurtner, 1985]. When using kinematic, pseudo-kinematic, static or reoccupation techniques utmost attention has to be paid to the correct handling of ambiguity parameters.

I : 3. Motivation

Encouraged by the performance of conventional static positioning with GPS the question was raised whether modified positioning techniques could be employed for detail surveys with competitive performance compared to classical surveying methods. A first analysis of existing positioning techniques with GPS led to the conclusion that ambiguity resolution represents the key issue considering detail surveying applications with GPS. Using either static or kinematic techniques, ambiguity resolution is the determining factor for performance. This was the primary motivation to analyze the ambiguity resolution process. The findings of this research work should then lead to a general ambiguity resolution technique for static and kinematic positioning techniques in small areas (up to 10 x 10 kilometers).

I : 4. Scope and Objectives

Following the primary goal of this work, namely to study the aspects of ambiguity resolution and to develop a general ambiguity resolution approach, the objectives are:

- Evaluate and validate state of the art positioning techniques with GPS as far as their suitability for detail survey applications are concerned.

- Study the ambiguity resolution process with respect to satellite geometry and systematic disturbances.

- Evaluate state of the art ambiguity resolution techniques and elaborate their advantages and disadvantages.

- Study adjustment and filter techniques as candidate data analysis mechanisms for a positioning method in detail surveys.

- Provide statistical means to assess results, accuracies and reliability measures.

- Develop a general ambiguity resolution strategy based on statistical considerations.

- Implement such a strategy as part of the Bernese GPS Software.

- Test and evaluate the implemented strategy regarding performance and reliability.

- Conduct a campaign to gather actual measurements in order to prove the validity of the proposed approach.

- Integrate the proposed ambiguity resolution approach into a surveying technique and conduct some initial tests.

I : 5. Chronicle and Summary

This work was started in 1987 with a survey of GPS literature focusing on data analysis techniques, ambiguity resolution strategies, adjustment techniques, filter techniques, and statistical hypothesis testing (the references to the related literature can be found in Appendix B). Detailed surveys of state of the art positioning techniques were carried out to evaluate the suitability of specific approaches for rapid positioning over distances up to 10 kilometers. These reviews showed clearly the key role of ambiguity parameters and especially the resolution of them to integer values. Surprisingly, there was hardly any study to be found in literature covering a detailed analysis of ambiguity resolution and its determining factors. Therefore, a detailed study of ambiguity resolution based on variance-covariance analysis techniques was our starting point. Variance-covariance analysis techniques were chosen in order to evaluate current as well as future satellite constellations. Sequential least-squares and Kalman-filter techniques were implemented as analysis tools on a PC environment. In addition, the POPS Software package [FREI et al., 1986] was modified so that satellite positions could be computed for currently available and future satellite constellations. These positions were then fed into the analysis tool, where, besides the formal accuracies, additional figures were computed e.g. the trace of the inverted normal equation system (Q_{XX}), the eigenvalues of Q_{XX}, some reliability indicators etc. Different measuring scenarios were simulated and analyzed using single or double frequency data. The first series of investigations was carried out using a five satellite constellation as actually available in Switzerland in 1988. Already these initial computations showed that ambiguity resolution from a theoretical point of view should be well possible with less than a quarter of an hour of observations. Wondering, what kind of an impact the fixing of a single ambiguity parameter would have on the remaining ambiguities, the analysis tools were amended in such a way, that a sequential ambiguity resolution could be simulated. As soon as the formal accuracy for a particular ambiguity parameter dropped below a selectable threshold then this parameter was handled in further computation steps as if it was known. Repeating the initial investigations, this time with the attempt to fix ambiguities as soon as their formal accuracies dropped below a 30 millimeter threshold (corresponding to 15 % of the 19 centimeter wavelength), it could be demonstrated that ten minutes of L_1 observations suffice to resolve ambiguities with a five satellite constellation using the conservative and simple resolution strategy outlined above. The usage of L_2 data as implemented for these simulations improved the performance by a factor of about $\sqrt{2}$ simply because of doubling the amount of measurement data.

As known from experience the satellite geometry is the most important factor for a fast and reliable ambiguity resolution for short baselines. Wondering what kind of results could be achieved with the full satellite constellation, we carried out a series of tests with a predicted constellation as can be observed after the full deployment of the GPS (about 1993). An eight satellite constellation was analyzed with the same technique. The main conclusion was, that two to three minutes worth of L_1 and L_2 data will be sufficient to resolve ambiguities using the simple threshold criterion (at 30 millimeters) for the resolution process. These fairly promising projections for the future were accompanied by a rather strong disappointment considering what could be achieved with the available satellite constellation.

It was then recognized that the implemented ambiguity resolution strategy for the initial simulations was not optimal, with regard to the usage of the actually available information. Subsequent studies resulted in a proposal for a strategy which makes use of the fact that every resolved ambiguity strengthens the remaining unresolved ambiguity parameters as well as the coordinates considerably. Initial tests using this improved strategy to analyze actual measurements demonstrated a much improved performance : two to three minutes worth of L_1 and L_2 measurements proved to be sufficient to resolve ambiguities to integer values for most satellite constellations.

Selected simulations and feasibility studies out of these initial investigations were presented at the "Fifth International Symposium on Satellite Positioning, March 13 - March 17, 1989, Las Cruces , New Mexico, USA". The paper was entitled "Some Considerations Concerning an Adaptive, Optimized Technique to Resolve Initial Phase Ambiguities for Static and Kinematic GPS Surveying Techniques". Part II of this document contains an edited version of the original paper presented at the Las Cruces conference.

Recognizing that the ability to resolve ambiguities is almost solely a function of the available satellite geometry, the question arises, whether this geometry could be improved by special surveying methods. Following an idea outlined by B. Remondi [REMONDI, 1988], namely to reoccupy one and the same survey site twice or more times within one to two hours for only a few minutes, a series of tests with real data was carried out. From a data analysis point of view each group of satellites was handled totally independently. A new set of ambiguity parameters was introduced for each observation group regardless if there were or were not identical satellites in these groups. Running these tests under varying scenarios (occupation time, time between occupations, number of satellites) led to the conclusion, that indeed ambiguity resolution was reliable using only a few measurements gathered during the short observation periods (one to two minutes). The basic idea behind such a reoccupation approach is that it is not necessary to remain on a particular survey site for one hour or longer; information of almost identical quality (phase noise) can be acquired in visiting the survey marker several times with short occupation times distributed over a longer period. The reduction of phase noise is anyhow limited by unmodelled systematic disturbances which vary with time. So, even observing continuously for several hours will not bring down the phase noise level below the two to three millimeter level. However, the change in the constellation of the satellites and hence the change in satellite geometry during this period provides the required strength of geometry to resolve the ambiguities successfully.

Reducing the site occupation time leads to a growth of the formal accuracies for the parameters to be determined (ambiguities, coordinates). This in turn complicates the ambiguity resolution in the sense, that more and more different combinations of integer ambiguities have to be analyzed in order to select the correct ones. The required computing power is not negligible (see e.g. [REMONDI, 1990]) and it clearly grows with the number of ambiguity combinations to be analyzed. In summary these early studies showed that a further analysis of the variance - covariance matrix was necessary in order to obtain efficient algorithms. The first action was to implement a general ambiguity search into the latest version of the Bernese GPS Software Package. A general search strategy represents the worst case as far as the required computing time is concerned. However, its main advantage is its reliability. All combinations of integer values within ambiguity-specific confidence ranges are evaluated in terms of resulting a posteriori variance of unit weight.

When dealing with very short observation periods, either for single or multi-occupation scenarios, an additional complication may occur because several integer combinations can yield almost identical a posteriori variances of unit weight. So, the correct solution has to be somehow selected based on objective selection criteria. In addition, such a selection and validation process should indicate whether the available information (primarily measurements) is sufficient to resolve the ambiguities or if additional information has to be provided for a successful resolution. Considering these findings from our initial research work, two questions got the full attention in further studies , namely:

(i) How can the information contained in the variance-covariance matrix for the estimated parameters (coordinates and real-valued ambiguity parameters) be fully utilized to speed up the ambiguity resolution process?

(ii) What kind of test methods can be employed to build objective selection and validation criteria to guarantee a reliable ambiguity resolution?

The starting point for the attempt to speed up the ambiguity resolution was the general search strategy. It can be seen as an evaluation of every grid-point in an n-dimensional parallelogram centered on the initial solution, where its size is given by the variances of the parameters estimated in the initial adjustment (coordinates and real-valued ambiguity parameters). The actual dimension of this hypercube is determined by a unique scale factor determined through statistical considerations regarding the desired confidence-level. Due to the fact, that correlations among the estimated parameters are not used, the resulting confidence region is big and as a matter of consequence more integer ambiguity combinations may have to be evaluated than would be theoretically required. As known from adjustment theory [VANICEK, KRAKIWSKY, 1986], [PELZER, 1985] the shape of such an n-dimensional confidence region, when considering the variances and covariances, is a n-dimensional hyperellipsoid. Therefore, only the grid-points located inside this hyperellipsoid would have to be evaluated. The determination of the grid-points inside this hyperellipsoid is not a trivial task. This is why our actual implementation was based on a slightly different approach. The search for grid points to be evaluated is carried out using one dimensional confidence ranges instead of the n-dimensional hyperellipsoid. They were built applying the error propagation law on functions of the estimated ambiguity parameters. Only integer ambiguity combinations which proved to be compatible with these confidence ranges were selected for subsequent determinations of the associated variances of unit weight. Initial tests have shown that this technique is capable to eliminate all the combinations of integer ambiguities which must not be considered for the final solution. The few remaining combinations have to be analyzed in terms of the resulting a posteriori variances of unit weight and subsequently, whether a unique solution exists or not. Data from the Turtmann 89 campaign was used [ROTHACHER et al., 1990] to test the potential of this search technique under various scenarios including single- and multi-occupation applications. These initial results were presented at the assembly of the International Association of Geodesy in August 1989 in Edinburgh, Scotland. Our knowledge at that time was, that typically two minutes worth of L_1 and L_2 data for 4 - 5 satellites were sufficient to resolve the ambiguities for baselines up to about 10 kilometers. The use of L_1 data only to 4 - 5 satellites enabled to resolve ambiguities with typically eight to ten minutes worth of data. The studies were continued with a survey of statistical hypothesis testing techniques with the goal to evaluate specific test approaches which can be used to decide whether the set of integer ambiguities yielding the smallest rms error a posteriori represents the correct solution and whether the smallest a posteriori variance of unit weight is compatible to its expectation value (a priori variance of unit weight). A χ^2- and an F-test were found to be suitable for these tasks. Special procedures were added to the initial ambiguity resolution technique to handle dual frequency data in an optimal way. From then on this approach was called "Fast Ambiguity Resolution Approach (FARA)". A detailed introduction to the concepts and techniques applied for the "FARA" is presented in the third part of this document with the title "Rapid Static Positioning based on the Fast Ambiguity Resolution Approach: Theory and Initial Results". An initial version of this paper was accepted for publication by "Manuscripta Geodetica" in August 1990.

In order to assess the potential of the "FARA" under different measuring scenarios with different types of measurements the "FARA 90" campaign was conducted in spring 1990. Ten baselines in a distance range from eight meters up to three kilometers were repeatedly measured with site occupation times in the order of five minutes. The tests were carried out with two WM102's and two Trimble 4000 SLD receivers. The data were analyzed using a prototype version of the "FARA" which was added to the Bernese GPS Software package running on a PC. The results obtained are presented in Part IV of this document. First results were presented at the "GPS'90" conference in Ottawa, Canada in September 1990 and the corresponding paper will be published in the symposium proceedings.

In general, the results are extremely encouraging for both, single and multi-occupation scenarios. It could e.g. be shown that one minute of L_1 and L_2 observations is sufficient to resolve ambiguities and hence provides sub-centimeter accuracy with a single occupation of a specific survey marker. This is true for receiver types acquiring the second frequency with the P-Code or a squaring technique. Considering detail survey applications, this performance is the key to a more than competitive surveying technique in comparison to classical detail surveying methods (e.g. total stations). Since the "FARA 90" campaign demonstrated the ability to perform positioning tasks within very short observation intervals the question is risen, how many measurements within what time interval actually are the absolute minimum allowing a successful ambiguity resolution. Processing selected baselines out of the "FARA 90" campaign with 45, 30 and 15 seconds worth of L1 and L2 data indicate, that an almost instantaneous ambiguity resolution should be possible provided a good satellite geometry is available (six or more satellites with a reasonable distribution over the sky).

The ideal situation would be real-time data processing. However, several demanding and complex tasks would have to be performed for such a real-time data analysis capability (data link, processing power in a field-worthy computer, etc.). In accepting several limitations and restrictions, such a real-time processing capability could be replaced by a tool which provides a prediction for ambiguity resolution performance. Such a tool could work on the basis of individual measurements and its tasks could include an assessment of quality for individual measurements. Following these ideas the basic theoretical considerations for such a prediction capability were elaborated and summarized in Part V of this document.

In summary the following four major topics were treated :

(i) The development of a generally applicable ambiguity resolution approach, called "FARA", which is based on a rigorous and optimal treatment of measurements and statistical data.

(ii) The demonstration of the potential of the "FARA" in terms of reliability and efficiency in analyzing different data sets under different measuring scenarios (single and multi-occupation scenarios).

(iii) The test of a rapid static positioning technique for short baselines based on the "FARA".

(iv) The proposal of candidate techniques which could serve as tools to predict ambiguity resolution.

The potential of the "Fast Ambiguity Resolution Approach (FARA)" is promising considering its possible exploitation for various surveying techniques with GPS including static, rapid-static and even kinematic techniques. All presented tests have shown that already with the currently available satellite constellation survey markers can be positioned with a single occupation sitting only for one minute on a particular site. Future constellations will even improve the performance so that an almost instantaneous ambiguity resolution will become reality.

Our emphasis so far was on theoretical considerations and on their exploitation for rapid static positioning techniques. There are additional areas of interest which need further attention. The two most attractive ones are the application of the "FARA" for medium range baselines and investigations in view of a utilization of the "FARA" as a technique to resolve ambiguities on the fly for true kinematic applications.

I : 6. Overview and Structure

This document is based on three papers which have been already published. These papers are :

(i) "Some Considerations Concerning an Adaptive, Optimized Technique to Resolve the Initial Phase Ambiguities for Static and Kinematic GPS Surveying Techniques". Presented at the Fifth International Symposium on Satellite Positioning, March 13 - March 17, 1989, Las Cruces, New Mexico, USA. Published in the Symposium Proceedings.

(ii) "Rapid Static Positioning based on the Fast Ambiguity Resolution Approach (FARA): Theory and First Results". Manuscripta Geodaetica, December 1990.

(iii) "Rapid Static Positioning based on the Fast Ambiguity Resolution Approach : The Alternative to Kinematic Positioning". Presented at the Second International Symposium on Precise Positioning with the Global Positioning System "GPS'90", Ottawa, Canada, September 3 - September 7, 1990. To be published in the Symposium Proceedings.

These papers are the kernel of Parts II, III, and IV of this thesis. In addition, this document contains the introduction as Part I, the prediction of ambiguity performance as Part V, a flow diagram of the Fast Ambiguity Resolution Approach "FARA" as Appendix A and references to related literature as Appendix B :

Part I	**:**	**Introduction**
Part II	**:**	**Simulations and Variance-Covariance Analysis**
Part III	**:**	**Fast Ambiguity Resolution Technique**
Part IV	**:**	**Measurements, Computations and Results**
Part V	**:**	**Prediction of Ambiguity Resolution**
Appendix A	**:**	**"FARA" : A Flow Diagram**
Appendix B	**:**	**References and Literature**

I : 7. Acknowledgements

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Erwin Frei
Im März 1991

Part II : Simulations with Variance-Covariance Analysis

Some Considerations Concerning an Adaptive, Optimized Technique to Resolve the Initial Phase Ambiguities for Static and Kinematic GPS Surveying - Techniques [1)]

1) A first version of this paper has been published in the proceedings of the "Fifth International Symposium on Satellite Positioning, Las Cruces, New Mexico, USA, March 1989".

Abstract

Tracking the carrier phases on the L_1 and L_2 signals would provide high - precision range measurements to GPS satellites if the so called initial phase ambiguities were resolved. The determination of these initial phase ambiguities represents the crucial element in high - precision positioning techniques with GPS. This has been the motivation to conduct a detailed study on the parameters which could possibly influence the resolution of these initial ambiguity parameters. Among the aspects to be studied the impact of geometry on the resolution characteristics has been given special attention. Apart from geometry, various other factors have been studied, e.g. the impact of atmospherical effects, the disturbances caused by multipath and the sensibility of the resolution process to external aids (e.g. distance measurements, approximate positions etc.).

II : 1. Introduction

W. Gurtner [GURTNER et al., 1985] has shown that resolving ambiguities improves the positioning accuracy by a factor of four even in the case where the observation time is long (several hours). This was the initial motivation to develop sophisticated methods to tackle the ambiguity resolution problem. Most of the analysis has been concentrating on selecting the correct set of initial ambiguities in a given observation scenario. The amount of data necessary and the observation time needed to resolve ambiguities safely has been treated as a secondary issue. As productivity and effectiveness have become primary requirements in utilizing GPS techniques for practical surveying the actual observation time needed to obtain the necessary accuracy has been given much more attention.

A new area in GPS technology has been opened with the proposal by B. Remondi [B.REMONDI, 1985] to utilize the Global Positioning System in a quasi-kinematic mode. The "piece de resistance" of this approach is the fact that GPS receivers are capable of tracking the phase to a particular satellite whilst the receiver is moving. Once the initial ambiguities have been solved, one of the participating receivers can move to the next site. Then, the position of this site can be determined in seconds if lock to the satellites has been maintained whilst the receiver was moving.

These techniques undoubtedly represent a tremendous potential for surveying applications. Especially, the quasi-kinematic technique seems to become a real competitor to classical surveying methods in detail surveying. Let us concentrate for a moment on the requirements of detail surveying with GPS. The main task is the determination of object-coordinates in the sub-centimetre accuracy-range. The coordinates themselves have to be known in a user-specified coordinate system rather than in a global reference frame like e.g. the World Geodetic System 1984 (WGS-84). Receiver separation distances for the area to be surveyed are typically of the order of about one to ten kilometers. To make use of the quasi-kinematic technique the phase ambiguities have to be resolved before the actual positioning procedure can be started.

Unfortunately, there are also limiting factors to the use of the quasi-kinematic technique for detail surveys :

(i) The technique is based on the assumption that the receivers maintain lock to the satellites tracked. Surveying in a built-up area will result once in a while in the loss of satellite-signals by accidental shading of the antenna. This requires a repeated determination of initial phase ambiguities. It takes typically 15 to 20 minutes to resolve ambiguities (L_1 only) unless additional information or special methods are employed to speed up the determination process. A few of the methods in use will be discussed later. However, losing satellites periodically degrades the performance of quasi-kinematic surveying almost to the performance of traditional static GPS techniques as far as productivity is concerned.

(ii) The lack of a data-link between the stationary and the roving receiver (at least for the time being) makes it very difficult to spot cycle slips or a possible loss of satellite signals during data gathering. Such problems will finally be detected in the post mission data analysis. The only hope for such situations are methods for a cycle slip repair. These methods tend to be complex and not reliable. So, in the worst case, the point-positioning cannot be performed as planned.

(iii) In order to avoid losses of lock the surveyor in the field would have to plan his movements from one site to another very carefully. This would severely restrict the user's mobility, which is one of the strongest points in surveying with GPS.

The question arises whether there are approaches to overcome a few of the problems outlined above. A potential method would be the combination of different sensor-types to bridge losses of lock to the satellites. Inertial systems are often mentioned in this respect, but their size, price and degree of complexity still seems not to favour an integration with GPS-receivers for detail survey applications.

Another solution is to speed up the initial ambiguity resolution process. If the observation time needed to resolve ambiguities would come down to two to three minutes one could substantially relieve the requirement for a "safe" path from one site to another. Thinking further along these lines leads logically to the question, whether it might become possible in the near future to resolve phase ambiguities almost in real time by means of high precision P - code measurements to the satellites. Although these ideas sound futuristic, present receiver technology seems already to point into that direction. However, already the reduction of the necessary observation time to resolve ambiguities from fifteen minutes to two or three minutes would set another highlight to surveying with GPS.

Based on these considerations a joint investigation was started by the Astronomical Institute of the University of Bern, Switzerland and Leica Heerbrugg Ltd., Switzerland in order to study firstly the feasibility of such an approach, and secondly to elaborate the basic requirements for a subsequent implementation. The main objective of this investigation was to learn more about the basics of ambiguity resolution as well as the sensitivity of ambiguities with respect to factors such as satellite geometry, phase noise, systematic effects, etc.. The investigations will result finally in a proposal for an optimized and adaptive approach to ambiguity resolution for static and kinematic GPS applications in small networks. The full range of tasks for such an investigation is not fully covered in this chapter. It has been decided to concentrate on a few crucial elements.

The chapter is split into five sections. Section two entitled as "Ambiguities", includes basic definitions, a review of ambiguity resolution approaches currently in use and a summary of requirements for an optimized, adaptive strategy. Section three deals with the sensitivity of ambiguity parameters with respect to satellite geometry and external aiding. Based on these results section four will present a proposal for an optimized, adaptive approach to ambiguity resolution in small networks. The fifth section presents the results of two tests, which have been compiled to demonstrate the feasibility of the proposed approaches. Section six summarizes the results and presents an outlook to further investigations.

II : 2. Ambiguities

2.1 Definition

In our approach we use the double-differenced phase observable. For our purpose it is sufficient to express it as :

$$dd(l_f) + v_f = dd(R) + dd(D^{ION}) + dd(D^{TROP}) + \lambda_f \cdot N_f^{nm}$$

(2.1.1)

where :

dd(...)	:	Double difference operator (stations A, B; satellites m, n),e.g., $dd(R) = (R_A{}^n - R_B{}^n) - (R_A{}^m - R_B{}^m)$: $R_K{}^i$ being the distance from station K to satellite i,
$dd(l_f)$	:	Double-differenced phase observation for frequency f (with f := 1,2),
v_f	:	Residual,
N_f^{nm}	:	Integer ambiguity,
R	:	Slant range to the satellite at observation time t,
D^{ION}	:	Ionospheric refraction,
D^{TROP}	:	Tropospheric refraction and
λ_f	:	Wavelength for frequency f.

In equation (2.1.1) we assume that both receivers are synchronized to GPS time within one microsecond or that this synchronization is known to within one microsecond.

The unknown parameters are the coordinates of the survey marker (implicitly contained in the slant ranges R to the satellites) and the ambiguity parameters N_f^{nm}.

The double-differenced phase observable (DDPO) has got two unique properties worth mentioning:

(i) The DDPO preserves the integer nature of initial ambiguity terms N_f^{nm}.

(ii) N_f^{nm} are constants in time as long as the receiver maintains lock to the satellites. Ambiguity resolution techniques are employed to determine the correct set of integer values for the ambiguity terms N_f^{nm} based on real-valued estimates.

2.2 Current Resolution Approaches

Several approaches have been elaborated and presented by various authors. Four of them will be briefly reviewed and evaluated as far as their capabilities in static and kinematic surveying are concerned. The four approaches are:

(i) The classic static approach by e.g. G. Beutler (BEUTLER et al., 1984) or Y. Bock (BOCK et al., 1986).

(ii) The P-code aided approach by e.g. R. Hatch (HATCH, 1986).

(iii) The antenna exchange- technique by B. Remondi (REMONDI, 1985).

(iv) A KALMAN filter approach.

2.2.1 The classic static approach

Classic approaches are typically based on a two step procedure. The two steps are:

(i) - Estimation of initial ambiguity parameters as real-values in a batch-type adjustment.

(ii) - Fixing initial real ambiguities to integer values based on their estimated formal accuracy or by general search strategies.
- Introduction of integer ambiguity parameters as known quantities into subsequent runs.
- Assessment of success by evaluation of the resulting sum of squared residuals.

A variety of methods has been developed to fix real-valued estimates to integer quantities. The differences between individual methods are to be found in whether statistical information is used to fix ambiguities or not, and in the number of n - tuples processed to evaluate the most probable set of integer quantities. A detailed description of these classic techniques may be found in (BEUTLER et al., 1984).

How rigorous and efficient these techniques are, is mainly dependent on the number of fixed-ambiguity-sets evaluated to make the final choice. A full search over all possible combinations is optimal as far as reliability is concerned but it is definitely not efficient. Evaluating only a few sets leaves the concern whether the correct set has been selected or not. These classic techniques could be improved considerably if the available statistical and geometrical information would be used in this process.

2.2.2 The P-code aided approach

R.Hatch's approach (HATCH, 1986) makes use of P-code measurements on both GPS frequencies. His approach is indeed very powerful and allows for a moving receiver. Unfortunately, P-code measurements on both frequencies are taken by a very limited number of receivers.

2.2.3 The antenna-exchange approach

The antenna-exchange technique proposed by B. Remondi (REMONDI, 1988) is a very elegant and fast approach to resolve initial ambiguities. Its main disadvantages are the restrictions put on operational flexibility. Antennas would have to be close together throughout the entire survey mission to guarantee a fast recovering if satellite signals are lost occasionally.

2.2.4 A KALMAN-filter approach

The basic idea behind this concept is to set up a bank of KALMAN-filters, each of them fed with a different set of integer ambiguity parameters. As more and more measurements become available, the filter containing the correct set of integer values can be selected by simply checking the statistical behavior. This approach is more or less a sequential implementation of the classic approach discussed earlier in the paper.

2.3 Specifications for a new approach

As the brief review of ambiguity resolution techniques above has shown, the aspects to be considered for the design of an adaptive, optimized ambiguity resolution technique are complex. An optimized technique should be

(i) fast,
(ii) reliable,
(iii) self-contained,
(iv) flexible,
(v) self-controlled and
(vi) automatic

Owing to the strong contradictions in what the qualities listed above require from such a technique, an optimum must be a compromise. The fact that a fast technique is not necessarily reliable illustrates nicely the dilemma.

A few of the qualities listed above need further explanation. The quality "self-contained" addresses the requirement for a complete technique, which handles all detail-survey requirements. Such an approach is flexible if different types of measurements and external information can be processed. The qualities "self-controlled" and "automatic" express the requirement for a technique which can be operated without user-interaction.

II : 3. Diagnoses

3.1 In General

Whether initial phase ambiguities can be resolved or not depends on a few rather complex factors, e.g. the satellite geometry, the type and quality of available measurements, the effects of systematic disturbances and so on. A thorough understanding of the impact of these parameters on the ability to resolve ambiguities is mandatory if a sound design of an optimized strategy is considered. It became clear however, that satellite geometry is the most important issue to be discussed in the next section.

3.2 Satellite Geometry

Variance-covariance analysis has been utilized as a primary tool for the evaluation of satellite geometry. This allows not only to evaluate present satellite constellations but also future constellations with more than the currently available seven satellites. The emphasis is put on results themselves rather on how they have been produced.

Two different satellite constellations were analyzed. Firstly, the constellation currently available at Heerbrugg, Switzerland in 1989 with five satellites and secondly, a predicted constellation as to appear at Los Angeles, California USA, in 1991 comprising eight satellites. The main objective of course is to study the impact on the ambiguity resolution process. Thus, a sequential least-squares approach has been implemented, which allows to extract the desired results, formal accuracy and correlation factors, for each individual measurement epoch. Not a single measurement had to be taken in order to generate these results. For all the different measuring scenarios only the satellite positions for each individual measurement epoch had to be determined.

Let us concentrate on a few interesting results of these simulations. Figure 3.2.1 shows the propagation of formal accuracies for the five-satellite-constellation at Heerbrugg in January 1989. L_1 measurements have been assumed to be taken every 60 seconds for a period of one hour. It has

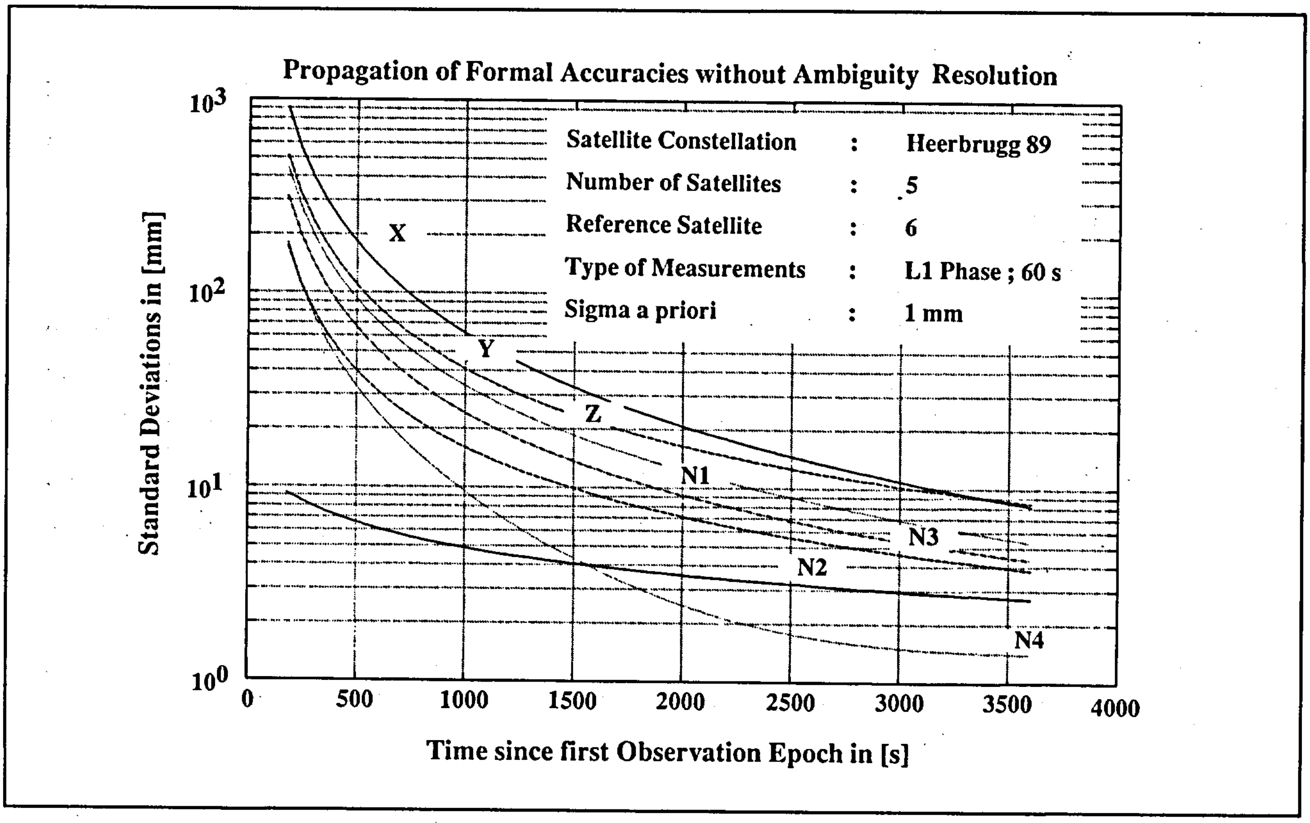


Figure 3.2.1: Propagation of Formal Accuracies without Ambiguity Resolution

been further assumed that the noise for every single phase measurement is one millimeter. Owing to the differencing of the original phase measurements correlations had to be introduced, which were rigorously treated to compile the subsequent Figures. The Y-axis denotes the standard deviations in millimeters for the unknown parameters. There are three coordinate-components X, Y and Z as well as four ambiguity parameters. Satellite six has been used as the reference satellite. The X-axis denotes the time in seconds past since the first measurement epoch. No attempt has been made to resolve ambiguities. Three interesting facts show up in Figure 3.2.1 :

(i) The standard deviations for different parameters at the same epoch vary rather strongly. These differences decrease with increasing time to about 10 mm after one hour observation time.

(ii) The ambiguity parameter N2 shows a formal accuracy below 10 millimeters already from the very beginning.

(iii) It takes 60 compacted one minute measurements to bring the formal accuracies for the coordinate components down to the sub-centimeter level

Point (i) is promising since a sequential process should allow to resolve one or another ambiguity parameter rather early in the process. This in turn should considerably improve the ability to resolve the remaining ambiguity parameters.

The excellent performance of the ambiguity-parameter N2 is astonishing. A closer inspection of the satellite constellation explains this fact. Satellites 6 and 11 (they form ambiguity N2) are close together in space. Remembering the observation equation (2.1.1) and considering two satellites with almost the same position in space allows an estimation of the ambiguity terms, but not the position.

With

$$\boxed{dd(R) \approx 0 \;;\; dd(D^{ION}) \approx 0 \;;\; dd(D^{TROP}) \approx 0} \tag{3.2.1}$$

equation (2.1.1) can be simplified to

$$\boxed{dd(l_f) + v_f = \lambda_f \cdot N_f^{nm}} \tag{3.2.2}$$

Ambiguities of this type can be resolved almost instantaneously considering the low phase noise of about two millimeters for a double-difference phase observation. Unfortunately, the result of such an early fix is not as one would desire. Because of the low contribution of such a measurement to the determination of the position, its fixing does not help too much to cure the weakness of the remaining ambiguity parameters.

If it would take 60 minutes to determine a position with subcentimeter accuracy, GPS would not make up a real competitive technique in detail surveying. It is well known, that GPS techniques can do much better in fixing ambiguities. Experience shows that 30 millimeters standard deviation (see Figure 3.2.1) represent a threshold, where a safe resolution of ambiguities can be expected. That is to say, as soon as an ambiguity parameter reaches the 30 millimeters accuracy level it can be fixed to an integer value with a high degree of confidence. Figure 3.2.2 shows the development of standard deviations, if ambiguities are fixed after having reached a formal accuracy of ≤ 30 mm.

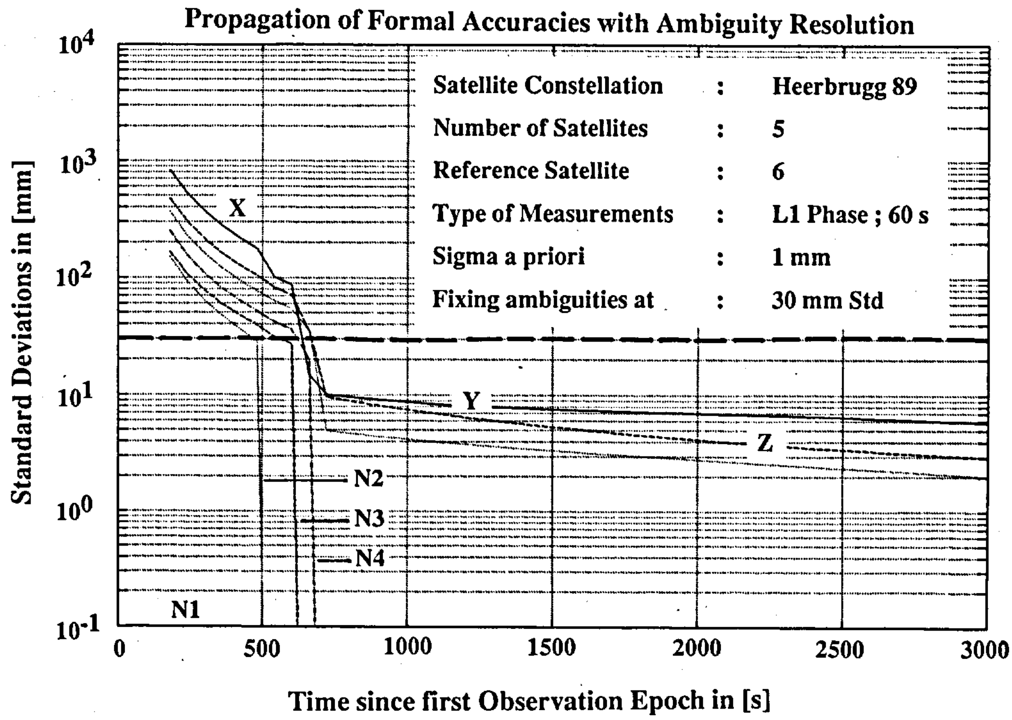


Figure 3.2.2: Propagation of Formal Accuracies with Ambiguity Resolution.

According to what can be read in Figure 3.2.2 it takes about 10 minutes to resolve all the L_1 ambiguities. As soon as ambiguities are resolved the coordinate accuracies fall below the 10 mm limit. From then on, accuracy improves very slowly.

So far, we were considering L_1 measurements only. The resulting improvement in adding the measurements from the second frequency is depicted in Figure 3.2.3 .

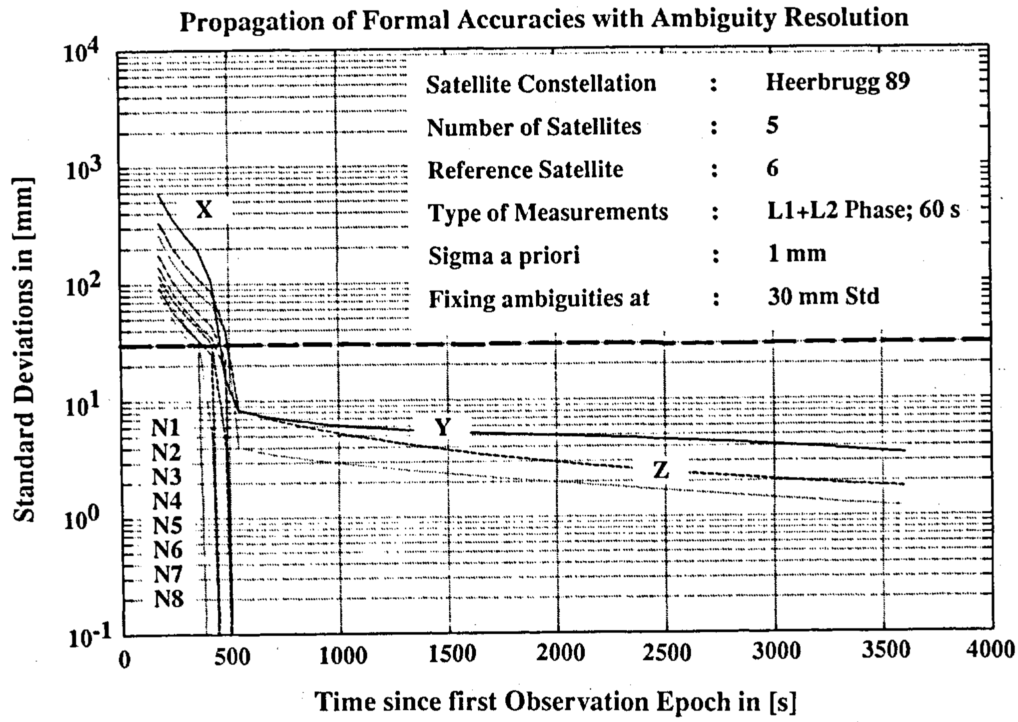


Figure 3.2.3 : Propagation of Formal Accuracies with Ambiguity Resolution for the Dual Frequency Case.

As to be seen in Figure 3.2.3 all ambiguities can be resolved within about 8 minutes and in addition, the formal accuracies could be improved by a factor $\sqrt{2}$ compared to the L_1 only case.

One might argue that formal accuracies are a nice, but rather unreliable instrument to forecast real-life situations. In expectation of such an argument real-life data has been processed to prove the validity of the presented results. Therefore, section five contains a brief summary of the results.

Figures 3.2.4 and 3.2.5 present the anticipated performance at Los Angeles in 1991 for the L_1 resp. the $L_1 + L_2$ case. The difference is evident : Taking L_1 measurements to eight satellites would allow us to resolve ambiguities within four minutes and in turn to obtain relative positions on the five millimeter accuracy level. Using in addition L_2 measurements would further improve the

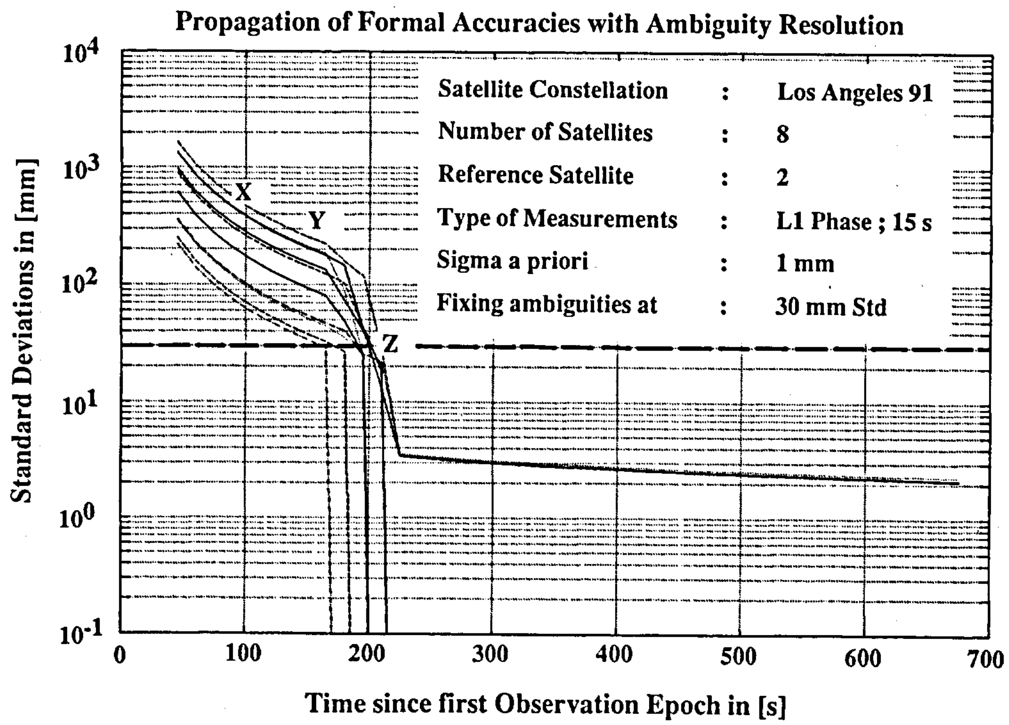


Figure 3.2.4: Propagation of Formal Accuracy for a Future Constellation using L_1 Measurements.

performance by one minute for the ambiguity resolution and by a factor √2 for the position accuracy. What a potential!

The performance in the above Figures does not make use of an optimized ambiguity search technique. The applied strategy is based on experience gained in processing real data. So, it could be assumed, that such a search strategy would be capable to further improve the performance. The question arises what such an optimized search strategy might look like and what level of gain in performance could be expected?

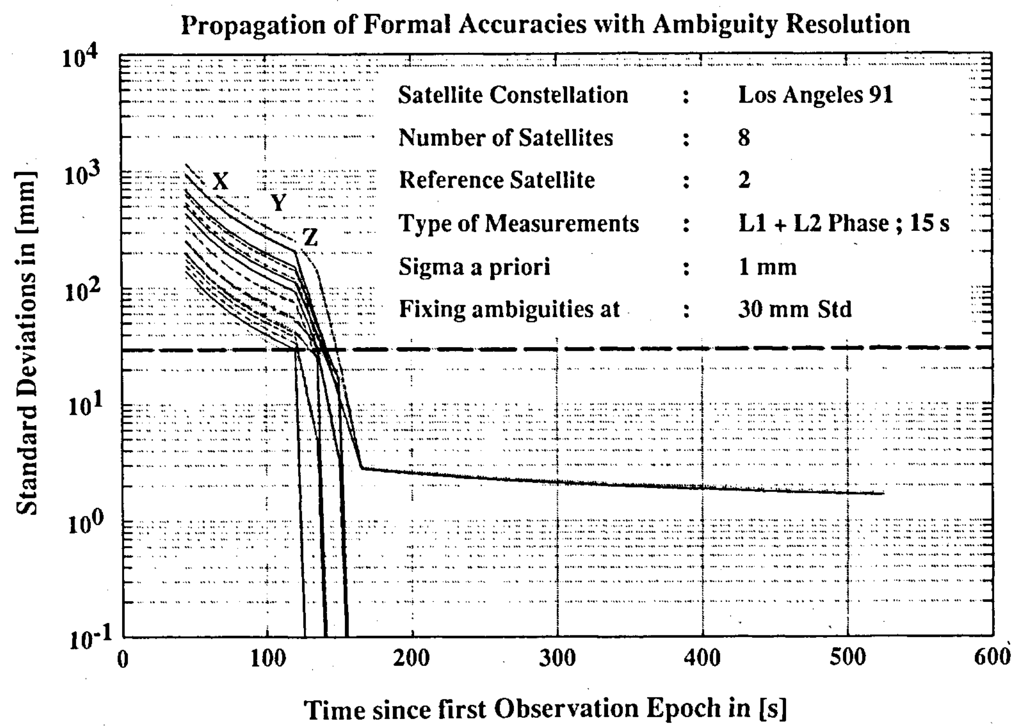


Figure 3.2.5: Propagation of Formal Accuracies for a future Constellation using L_1 and L_2 measurements.

II : 4. An Optimized Approach

As stated earlier, it is not intended in this chapter to present a search-strategy ready for implementation, but to discuss the essential elements of such an approach. What are now the essential characteristics of an adaptive, optimized technique?

(i) Differential GPS processing techniques have to become truly sequential processes capable of evaluating the performance and the probability of fixing ambiguities after each measurement epoch.

(ii) Statistical information has to be fully employed to decide, on the run, about the search strategy to be applied.

(iii) The selection of reference satellites has to be based on geometrical and statistical considerations rather than on satellite order or number of available measurements.

(iv) Information describing the expected performance and the measuring scenario has to be used to adapt and steer the search strategy.

(v) Indicators have to be elaborated which allow a reliable assessment of success.

(vi) Multiple solutions are not considered as final. In the case, where more than one solution seems statistically appropriate, additional information (observations and/or external information) has to be provided to make the final decision.

On very short baselines (≤ 5 km) the contribution of ionospheric and tropospheric refraction to the budget of unmodelled systematic errors can be neglected. Thus, the remaining systematic errors are to be explained mainly by the different electronic behavior of the two participating receivers. Contributions of that kind are usually small. So, one can expect the ambiguity determination to be driven uniquely by satellite geometry. Based on these considerations a search approach might contain the following steps:

(i) Select the double differenced ambiguity parameter with the smallest standard deviation. All possible satellite combinations have to be included in this search.

(ii) Determine the integer numbers within a band of plus and minus three times the standard deviation around the real-valued estimate of an ambiguity considered. If there happens to be only one single integer in this interval, fix the corresponding ambiguity.

(iii) Evaluate each integer ambiguity individually by introducing it as a known quantity into subsequent adjustment runs.

(iv) Select the integer ambiguity yielding the smallest a posteriori variance of unit weight. If there are several ambiguities yielding comparable a posteriori variances of unit weight, take the solution showing the smallest change to the previous solution vector. The alternative solutions may have to be checked subsequently.

(v) Proceed with step (i) as long as there are still ambiguities to be fixed. If a sudden change is observed in the a posteriori rms, return to the previous ambiguity and fix it to an alterative value.

Although not very sophisticated, this approach shows promising results when applied to real data.

II : 5. Feasibility Tests

In order to get an idea how much real data is required as a minimum for the resolution of ambiguities and in addition to check the feasibility of the proposed search strategy, three tests have been carried out with WM102's including baselines from eight meters up to seven kilometers. The receivers were recording one measurement per minute per satellite. The eight meter baseline was observed at Heerbrugg, Switzerland under a five-satellite-constellation. The longer baselines were observed in Sacramento, Cal., USA. The data were analyzed to answer the following questions:

(i) How long does it take to resolve ambiguities with L_1 measurements only?

(ii) How long does it take with both, L_1 and L_2 data?

(iii) Can the time needed to resolve ambiguities be shortened in applying the proposed search strategy?

(iv) How is the accuracy compared to ground truth for the different computation?

Table 5.1 below shows a summary of the results.

First Test :

As the computations show, it was possible to resolve ambiguities with two minutes of L_1+L_2 observations by applying a very simple search strategy under a five-satellite-constellation. The comparison with ground truth yields differences in the order of 1 to 5 millimeters.

Obs. time	Number of Sats	Classical Resolution using L_1	Classical Resolution using L_1+L_2	Proposed Search using L_1+L_2	Resulting Chord Length	Difference to Ground Truth
[min]					[m]	[mm]
18^m	4	●	●	○	8.159	7
8^m	4	●	●	○	8.144	8
7^m	5	●	●	○	8.152	0
6^m	5	×	●	○	8.150	2
5^m	5	×	×	●	8.150	2
4^m	5	×	×	●	8.151	1
3^m	5	×	×	●	8.147	4
2^m	5	×	×	●	8.151	1

Table 5.1: *Results from Heerbrugg Test (Legend : ● stands for a successful attempt to resolve ambiguities to integers, × stands for an attempt to fix ambiguities which failed and ○ indicates the cases where no attempt has been undertaken to fix ambiguities)*

The second test has been carried out with a baseline of about 1.5 kilometers in the Sacramento, Cal. area with a six-satellite-constellation. Table 5.2 summarizes the most important results.

Six samples of compacted one-minute L_1 measurements are sufficient in this case to resolve ambiguities. Only four samples are required if L_1 and L_2 data is available. If dual frequency measurements are available, the time required can be reduced to two minutes in utilizing the proposed search strategy. Owing to the lack of a ground truth, the resulting chord lengths using all available data has been compared. The agreement is again excellent.

In addition, a seven kilometer baseline in the Sacramento area was analyzed. Resolution of L_1 ambiguities has been possible after 14 minutes worth of data, whereas 9 minutes worth of data were sufficient to resolve L_1 and L_2 ambiguities together. More sophisticated search strategies than the proposed one would have to be employed to further reduce the time to resolve ambiguities. A

Obs. time	Number of Sats	Classical Resolution using L_1	Classical Resolution using L_1+L_2	Proposed Search using L_1+L_2	Resulting Chord Length	Difference to Session Solution
[min]					[m]	[mm]
46^m	6	●	●	○	1570.638	Ref.
6^m	6	●	●	○	1570.637	1
4^m	6	×	●	○	1570.636	2
3^m	6	×	×	●	1570.635	3
2^m	6	×	×	●	1570.634	4

Table 5.2: *Results from Sacramento Test (Legend : ● stands for a successful attempt to resolve ambiguities to integers, × stands for an attempt to fix ambiguities which failed and ○ indicates the cases where no attempt has been undertaken to fix ambiguities)*

comparison of chord lengths from the L_1 and L_1+L_2 computations yielded differences of 7 millimeters for the L_1 case and 1 millimeter for L_1+L_2.

II : 6. Conclusions

The simulations and tests described have been conducted in order to investigate whether or not it is possible to resolve ambiguities in a few minutes rather than in tens of minutes. An evaluation of formal accuracies has shown that it should be possible to resolve ambiguities in a few minutes if only geometry would have to be considered. Based on the study of formal accuracies a simple ambiguity search strategy has been proposed especially tailored to very short baselines. In order to assess the feasibility of the proposed approach three tests have been carried out with real WM102 data. Baselines of 8 meters, 1.5 and 7 kilometer have been analyzed. Two minutes of data were sufficient to resolve L_1 and L_2 ambiguities on two short baselines. Nine minutes were necessary on the 7 kilometer line. The number of simultaneously available satellites and the measurement quality seem to be the key elements for a fast ambiguity resolution on short baselines. At least five satellites are required to achieve the described performance.

Simulations with the future GPS satellite constellation have shown that it can be expected to resolve ambiguities within 30 to 120 seconds when eight satellites can be observed simultaneously. Undoubtedly these results are encouraging. However, more needs to be done to fully understand the mechanisms of fast ambiguity resolution.

Part III : Fast Ambiguity Resolution Approach

Rapid Static Positioning based on the Fast Ambiguity Resolution Approach : Theory and Initial Results [1)]

1) A first version of this chapter has been published in "Manuscripta Geodaetica" in 1990.

Abstract

The resolution of initial phase ambiguity parameters was recognized to be the key to sub-centimeter position accuracy in surveying with the GPS. The most popular method so far utilizing this fact is "kinematic-positioning" initiated by B. Remondi (REMONDI, 1986). It is based on the assumption that the phase-lock to all satellites can be maintained while moving from one survey marker to the next. If this can be achieved the resolution of ambiguity parameters is only required once at the very beginning of a survey. Then the occupation times for all subsequent points to be surveyed will be very short since the ambiguity parameters are already known. In our approach to "rapid static positioning" we avoid any assumptions concerning the phase lock while moving from one survey marker to the next. So does B. Remondi in his pseudo - kinematic positioning technique (REMONDI, 1988). We optimize the process of resolving initial ambiguity parameters by utilizing a search strategy which makes use of the full information contained in the variance-covariance matrix of the initial differential position adjustment (prior to ambiguity resolution). We show that the time required to resolve ambiguity parameters with this technique highly depends on the number of satellites observed, that it takes one minute to resolve ambiguities if 5 and more than 5 well distributed satellites can be observed with a single site occupation. Occupying the same survey marker twice within 1 - 2 hours for about 1 - 2 minutes is sufficient to resolve ambiguities if there are only 4 - 5 satellites available. In addition, we show that there is a big advantage in using dual frequency data rather than only single frequency data. One minute worth of L_1 and L_2 measurements are sufficient to perform a point positioning with a single site occupation. Results are presented using data stemming from classical static positioning campaigns.

III : 1. Introduction

The positioning accuracy for short baselines drops instantly below the sub-centimetre level as soon as the initial ambiguity parameters are known. Therefore, the time required to achieve accuracies in the order of 1 centimeter is solely determined by the time it takes to resolve the initial phase ambiguity parameters to integer values.

1.1 Classic Static Positioning

Early ambiguity resolution techniques have been primarily designed for a reliable determination of those parameters. The site occupation time and the number of measurements necessary to perform this task have been treated as a secondary issue. The receivers have normally been kept on a single survey marker for at least one hour or even longer. For detail survey applications, this technique can not be recognized as a real competitor to the classical equipment currently in use (total stations). This positioning technique with GPS, referred to as classical static positioning, is primarily a tool for applications with highest accuracy requirements.

For later comparisons some of the techniques to resolve initial phase ambiguity parameters when processing static data will be briefly reviewed. This discussion intends to present the underlying principles rather than actual implementations.

Most of the techniques are based on a two step approach : In the first step differential coordinates and the real-valued ambiguity parameters are estimated. The second step consists of strategies and algorithms to resolve the initial phase ambiguity parameters to integer values. Let us summarize three important techniques :

(i) Round the real-valued ambiguity parameters N_r ($r = 1,\ldots$) to the nearest integer value (Fig. 1.1).

(ii) Use the estimated standard deviation (m_{Nr}) to evaluate whether or not the resolution to an integer value is feasible from a statistical point of view. The ambiguity will be fixed if there is only one integer value within the confidence interval (app. $\pm$ 3 m_{Nr}) around N_r (Fig. 1.2).

(iii) General search : Form every feasible combination of integer ambiguity parameters around the real-valued estimates considering all ambiguity parameters simultaneously (Fig. 1.3). The set of integer ambiguities yielding the smallest standard deviation in a subsequent adjustment run is taken as the final solution.

```
          Ni<-- Nr
-|--------|--------♦--*-----|--------|--------|--
 3        4        5        6        7        8
```

Fig. 1.1 : *Rounding ambiguities to nearest integers ("*" represents the location of the real valued estimate N_r and "♦" represents the nearest integer ambiguity)*

```
        Ni<-- Nr
-|--------|--------|--*-----|--------|--------|--
 3        4        5 |      6        7        8
               <-♦-+--->
               ± 3 • mNr
```

Fig. 1.2 : *Use of the m_{Nr} to support the resolution of integer ambiguity parameters ("*" represents the location of the real valued estimate N_r and "♦" represents the nearest integer ambiguity)*

N_{1r}

```
-|--------|--------|------*-|--------|--------|--
7        8        9      | 10       11       12
```

N_{11} | N_{12}

```
<---♦-----+♦-------♦-->
```

$\pm 3 \cdot m_{N1r}$

N_{2r}

```
-|--------|------*-|--------|--------|--------|--
2        3       | 4        5        6        7
```

N_{21} N_{22} | N_{23} N_{24}

```
<-♦------♦----+-♦------♦---->
```

$\pm 3 \cdot m_{N2r}$

. . .

N_{nr}

```
-|--------|--------|--*-----|--------|--------|--
-2       -1        0 |      1        2        3
```

N_{n1} N_{n2} | N_{n3} N_{n4}

```
<----♦-------♦-+----♦------♦->
```

$\pm 3 \cdot m_{Nnr}$

Fig. 1.3 : Forming combinations of integer ambiguity parameters ("" represent the locations of real valued estimates N_r and "♦" represent the integers to be considered)*

A comparison of the three techniques outlined above shows that there are differences in the level of sophistication and in the amount of computations required to obtain the result. It is important to realize that not the full statistical information provided by the initial adjustment is used in the techniques above. Although the standard deviations for the ambiguity parameters are used to define the search ranges (strategies (ii) and (iii)), the correlations between parameters are not taken into account in the resolution process at all. As far as strategy (iii) is concerned, not using these correlations results in a suboptimal search. Suboptimal because the number of combinations of integer ambiguity parameters to be evaluated is much bigger if only standard deviations are used to define the search ranges instead of using in addition the given correlation information.

Different sets of integer ambiguity parameters lead to different positions. A resulting position can be taken as a valid alternative to the position estimated prior to ambiguity resolution as long as the two positions are in statistical agreement (see 3.2). No precautions are taken in the techniques presented above to avoid position determinations which are not in statistical agreement with the initial estimate of the differential position.

These general considerations point into the direction of potential improvements for the ambiguity resolution strategies used in static positioning techniques.

Several such improved techniques have been published lately by various authors e.g the techniques by Dong and Bock (DONG and BOCK,1989), G. Blewitt (BLEWITT,1989) and B. Remondi (REMONDI,1990). Dong and Bock propose a technique where resolving ambiguities on shorter baselines in a network allows to "bootstrap" ambiguity resolution to longer and longer baselines. Whether a single ambiguity can be resolved or not is determined by a two-dimensional decision region. This decision region is built as a function of closeness to the next integer and of some statistical considerations. The approach taken by G. Blewitt processes undifferenced data and then forms double-difference ambiguity estimates. The variance-covariance matrix is used to select the set of ambiguities (double-differenced biases) which are theoretically best determined (optimal double-differencing transformation). B. Remondi's approach is based on some sort of a general search technique referred to as "Ambiguity Function Method (AFM)". Alternative positions in the neighborhood of an initial triple difference solution are processed with the AFM to find the position which is rated highest in evaluating all the single differences for all the alternative positions.

1.2 Kinematic Positioning

B. Remondi's positioning technique [REMONDI, 1986], referred to as kinematic-positioning, has been recognized as an elegant and efficient tool for positioning with the GPS. This approach relies technically on the assumption that the receiver does not loose lock to the satellites while moving from one survey marker to another. The position of these survey markers can then be determined within seconds. This is possible due to the fact that the initial phase ambiguity parameters, determined at the very beginning of the survey, remain valid as long as no loss of lock occurs. If the receiver looses lock to the majority of satellites, the phase ambiguity parameters have to be re-determined. Surveying in urban areas will cause the receiver once in a while to loose lock to the satellites. Whenever this happens, new initial ambiguity parameters have to be determined. As a consequence, the positioning productivity decreases considerably.

In consideration of these facts we conclude that the kinematic-positioning technique depends strongly on a fast and efficient ambiguity resolution technique either to resolve the ambiguity parameters at the very beginning of a survey or whenever a loss of lock occurs.

One way to tackle the problem of resolving initial ambiguity parameters is to first set up the antennae over known survey markers. This allows for an almost instantaneous determination of these ambiguity parameters. Unfortunately, there are not always known survey markers available when the receiver loses lock to the satellites in the middle of a survey.

A second approach to solve our problem is the antenna-swapping technique initiated by B. Remondi [REMONDI, 1988]. Its main disadvantages can be seen in the restrictions put on the operational flexibility, since the antennae have to be close together during the entire survey campaign to guarantee a fast recovering of ambiguity parameters. From an operational point of view this is definitely not an optimum considering the fact that two field groups are required to handle the two participating receivers.

An often recommended procedure to recover the ambiguities is to return to the last successfully determined survey marker. Only a few seconds worth of data suffice to recover the initial phase ambiguities (as for the case where known survey markers are available). This approach is very efficient if there is actually a safe path around the obstruction which caused the loss of lock in the first place. If not, because of bridges, tunnels, buildings, these hurdles have to be taken before a new ambiguity resolution can take place.

In summary, the kinematic positioning technique is definitely a powerful and elegant approach to rapid positioning. However, to make this approach a fully operational and efficient tool for the practical surveyor requires a fast and reliable resolution of initial ambiguity parameters as well as a quick method to recover the integer ambiguity parameters after a loss of lock.

1.3 Code and Phase Techniques

Alternative approaches to rapid positioning have been presented by R. Hatch [HATCH, 1986 , G. Wübbena [WUEBBENA,1985], and recently by G. Blewitt [BLEWITT et al.,1989]. They are based on the use of simultaneous phase and P-code measurements taken on both GPS frequencies (L1 and L2). This requires firstly, high precision dual band P-code (and of course phase measurements) and, secondly, a wide and narrow lane ambiguity resolution technique. It allows resolving the initial phase ambiguities within a few tens of seconds. The only disadvantage can be seen in the fact that high precision P-code measurements on both GPS carriers are required. These measurements are not always available for all users because of US-governmental regulations.

1.4 Our Approach

The review of the above methods led to the following specifications for a fully operational positioning technique :

(i) It has to guarantee that each individual survey marker can be determined reliably and within the specified accuracy.

(ii) The position determination of a specific marker should not depend on what has happened before and after it was observed. This requires in turn that the coordinates and the ambiguity parameters can be estimated using only the measurements taken on this specific survey marker.

(iii) The site occupation time must be short (of the order of 1 minute).

The classical static positioning technique presented above meets the first two requirements but it takes a few tens of minutes rather than a few tens of seconds to resolve the initial ambiguities.

Initial investigations have shown (FREI and BEUTLER, 1989) that ambiguity parameters can be resolved within a few minutes if 7 to 8 satellites are observed simultaneously. Fortunately, the full GPS constellation will provide these 7 to 8 satellites for certain periods of time. The question remains what can be achieved with 4 to 6 satellites.

Our strategy, which we call "Fast Ambiguity Resolution Approach (FARA)", uses the statistical and geometrical information provided by the initial differential position adjustment (where receiver coordinates plus real-valued ambiguities are estimated). It represents the key for several positioning methods e.g. classical static, kinematic, pseudo-kinematic etc. Its main characteristics :

(i) The basis is an initial adjustment (coordinates and real-valued ambiguity estimates) yielding the solution vector, the associated variance-covariance matrix and the a posteriori standard deviation of unit weight.

(ii) A search strategy is used for ambiguity resolution. It is based on concepts from statistics and generates the valid integer ambiguity combinations to be analyzed in subsequent adjustment runs. Statistical criteria are used to select the final solution(s).

(iii) The search range for each individual ambiguity is defined by the variance-covariance matrix of the initial adjustment. The search technique is adaptive in the sense that the number of ambiguity combinations to be processed depends on the quality of the initial estimates.

(iv) The technique does <u>not</u> depend on the selection of the reference satellite.

(v) It is self-contained. The procedure does not need other information than the ones listed under (i) to determine the final solution.

(vi) The technique automatically decides whether the information used in the initial adjustment suffices to resolve the ambiguities to integer values or not (for the case where multiple solutions exist).

The occupation time required depends on the number of satellites and their distribution in space. We will see that occupying the same survey marker more than once (BEUTLER, FREI et al.,1989) within one to two hours for only one to two minutes is sufficient to determine the relative position with sub-centimetre accuracy (assuming antenna separation distances not longer than 10 kilometers). For this positioning technique satellite constellations with only four satellites may be used. If dual band receivers are available, the positioning can be done even more rapidly. Relationships between L1 and L2 measurements reduce the number of ambiguity combinations to be processed considerably.

Satellite geometry and its impact on the positioning performance is discussed in the next section. Then the theoretical aspects of our approach will be presented. Results from processing static data illustrate operational aspects and achievable accuracy using our approach.

III : 2. Satellite Geometry

The studies by (FREI and BEUTLER, 1989) revealed that future constellations with eight and more satellites will enable to resolve ambiguity parameters using very short occupation times. Figure 2.1 shows the development of formal accuracies of the unknown parameters (station coordinates and ambiguities) for such an eight-satellite-constellation. The criterion applied to fix a particular ambiguity parameter to an integer value in this simulation is rather simple, somewhat arbitrary and not an optimum : According to strategy (ii) described above ambiguity parameters were fixed as soon as their formal accuracy became smaller than 30 millimeters. No search strategies were used to speed up the ambiguity resolution process. We see that even with this simple approach ambiguity resolution was completed after approximately three minutes.

Figure 2.2 shows the development of formal accuracies for the unknowns for a four-satellite-constellation presently available. The time required to resolve ambiguities for this scenario is rather long compared to the constellation above. The performance is obviously a function of the number of available satellites. The question arises whether this lack of geometrical strength in the four satellite constellation can be compensated by means of a specific surveying methodology and still keep the requirement of minimum site occupation time. It is known from static positioning that observing four satellites for one hour or even longer in most cases allows for ambiguity fixing on short baselines. Gathering measurements over a longer period serves mainly two purposes: Firstly, systematic disturbances which vary in time, like e.g. ionosphere or multi-path, can be strongly reduced or might even average out. Secondly, the quality of the determination can be improved due to the change of the observed satellite geometry over the measuring time. A reduction of measurement noise however is not required thanks to the high quality of individual phase measurements. Considering the special case of short baselines leads to the speculation, that only a few data samples distributed over, let us say, a one hour period might be sufficient to resolve the ambiguity parameters and therefore achieve the desired positioning accuracy. Processing real data has shown (BEUTLER, FREI et al, 1989) that just a few data points at the very beginning and at the very end of such a measuring period are sufficient to fix the real valued ambiguities to integers. The ambiguity parameters in such a determination can be treated in three different ways:

(i) One common set of ambiguity parameters is introduced for both observation groups (number of unknowns = number of satellites - 1).

(ii) One set of ambiguity parameters is introduced for the first set of observations and a second set of unknowns for the ambiguity differences of the second observation group with respect to the first group (number of unknowns = 2*(number of satellites - 1)).

(iii) A totally independent set of ambiguity parameters is introduced for each observation group (number of unknowns = number of satellites in first group + number of satellites in the second group - 2).

Scenario (i) demands for cycle-slip-free data and can therefore be seen as a kinematic technique. Scenario (ii) estimates possible cycle slips between the first and the second observation group. Therefore the receivers could be switched off between site occupations. It is assumed that the identical set of satellites will be tracked again in the second observation group. This scenario represents clearly a static technique. Scenario (iii) goes one step further by no longer assuming, as in scenario (ii), identical sets of satellites in both observation groups. Even if identical satellites can be observed in the second observation group they can be treated totally independently from the ones in the first group. Provided enough time has elapsed since the measurements of the first group such a "Two times four" satellite constellation is similar to an eight satellite constellation as far as the positioning performance is concerned. The only difference is the number of ambiguity parameters for the two constellations (7 unknowns for a proper 8 satellite constellation; 6 unknowns for the "two times four" constellation).

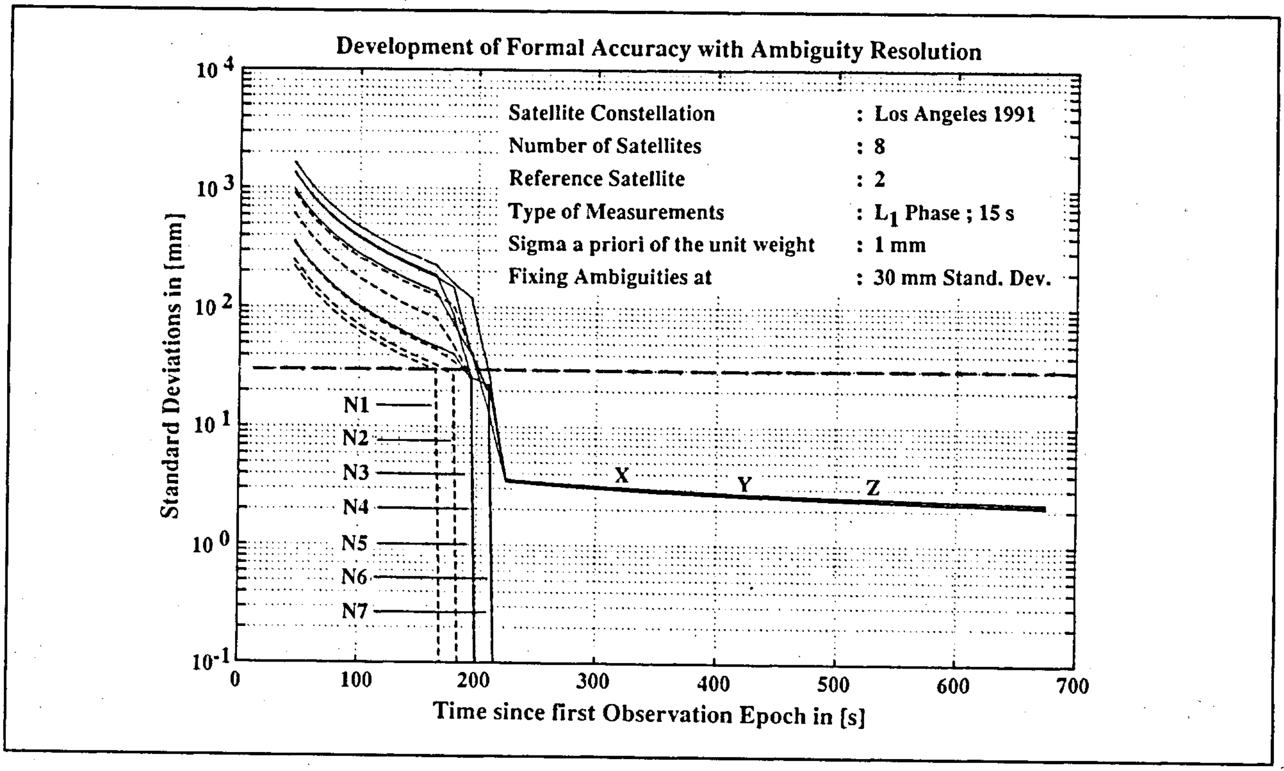


Fig. 2.1 : *Development of formal accuracies for an eight satellite constellation*

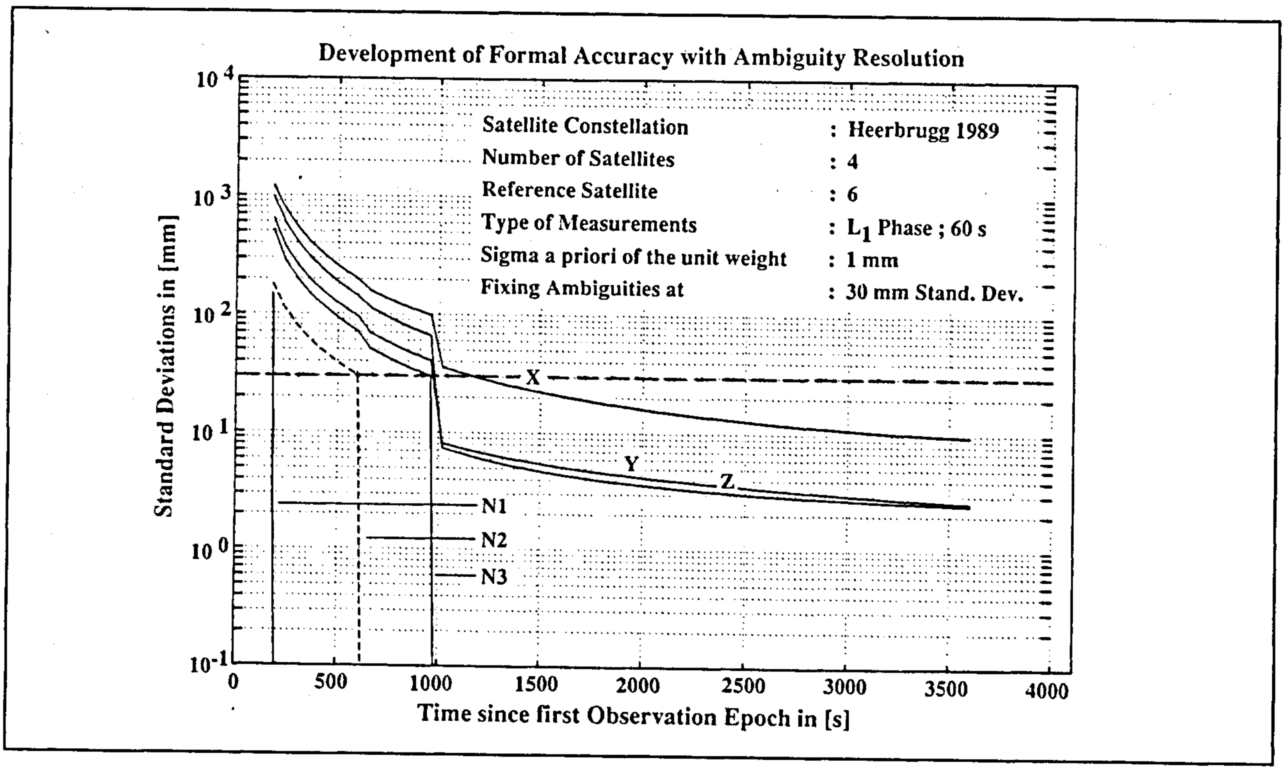


Fig. 2.2 : *Development of formal accuracies for a four satellite constellation*

III : 3. Fast Ambiguity Resolution Approach

3.1. Initial Solution without Ambiguity Fixing

In our approach we use the double-differenced phase observable. It can be expressed as :

$$dd(l_f) + v_f = dd(R) + dd(D^{ION}) + dd(D^{TROP}) + \lambda_f \cdot N_f^{nm} \qquad (3.1.1)$$

where :

dd(...)	:	Double difference operator (stations A, B; satellites m, n),
$dd(l_f)$	:	Double-differenced phase observation for frequency f (with f := 1,2),
v_f	:	Residual,
N_f^{nm}	:	Integer ambiguity,
R	:	Slant range to the satellite at observation time t,
D^{ION}	:	Ionospheric refraction,
D^{TROP}	:	Tropospheric refraction and
λ_f	:	Wavelength for frequency f.

In equation (3.1.1) we assume that both receivers are synchronized to GPS time within one microsecond or that this synchronization is known to within one microsecond.

The unknown parameters are the coordinates of the survey marker (implicitly contained in the slant ranges R to the satellites) and the ambiguity parameters N_f^{nm}.

The functional model for the adjustment (GAUSS - MARKOV) in its final form can be expressed by (for a detailed description see (PELZER,1985)):

$$\hat{L}_k = \varphi(\hat{X})$$
$$L_0 = \varphi(X_0)$$
$$\hat{L}_k = L_0 + \hat{l}_k$$
$$\hat{X} = X_0 + \hat{x}$$
$$\hat{l}_k = l_k + v_k = A_k \hat{x}$$

(3.1.2)

where :

X_0 : Approximate values for the unknown parameters,

$\hat{X}$: Least-squares estimate of the unknown parameters

$\hat{x}$: Least-squares estimate of the corrections to the approximate parameters

L_0 : Approximate values for the observations,

$\hat{L}_k$: Least-squares estimate of the observations at epoch k,

$\hat{l}_k$: Least-squares estimate of the corrections to the approximate observations at epoch k,

l_k : Corrections to the approximate observation at epoch k,

v_k : Residuals and

A_k : First design matrix at epoch k,

with the stochastic model :

$$K_{llk} = \sigma_0^2 \cdot Q_{llk} = \sigma_0^2 \cdot P_k^{-1}$$

(3.1.3)

where :

K_{llk} : Variance-covariance matrix for the observations at epoch k,

σ_0^2 : a priori variance of unit weight,
Q_{llk} : Matrix of cofactors for the observations at epoch k and
P_k : Weight matrix for the observations at epoch k.

All mathematical correlations introduced in forming the double-differenced phase observables are rigorously treated, so the matrices in (3.1.3) above are fully populated.

The normal equation system after j observation epochs is given by :

$$\left[\sum_{k=1}^{j} (A_k^T P_k A_k) \right] \cdot \hat{x}_j = \sum_{k=1}^{j} (A_k^T P_k l_k) \tag{3.1.4}$$

After j observation epochs the least-squares estimate of the corrections to the approximate parameters $\hat{x}_j$ are determined by :

$$\hat{x}_j = Q_{xxj} \cdot b_j \tag{3.1.5}$$

where :

$$Q_{xxj} = \left[\sum_{k=1}^{j} (A_k^T P_k A_k) \right]^{-1} = N_j^{-1}$$

$$b_j = \sum_{k=1}^{j} (A_k^T P_k l_k) \tag{3.1.6}$$

and

N_j : Normal equation matrix after j epochs,
Q_{xxj} : Matrix of cofactors for the unknown parameters (not yet scaled by the variance factor to obtain variances).

The a posteriori variance factor m_{0j}^2 (variance of unit weight) is given by :

$$m_{0j}^2 = \frac{\mathbf{v}^T\mathbf{P}\mathbf{v}}{\mathbf{n-u}} = \frac{\mathbf{v}^T\mathbf{P}\mathbf{v}}{\mathbf{f}} \tag{3.1.7}$$

where :

$$\mathbf{v}^T\mathbf{P}\mathbf{v} = \left[\sum_{k=1}^{j} (\mathbf{l}_k{}^T\mathbf{P}_k\mathbf{l}_k) \right] - \hat{\mathbf{x}}_j{}^T\mathbf{N}_j\hat{\mathbf{x}}_j \tag{3.1.8}$$

and

$$n = \sum_{k=1}^{j} n_k \tag{3.1.9}$$

Also :

$$m_{xji} = m_{0j} \cdot ([Q_{xxj}]_{ii})^{1/2} \; ; \; i = 1,...,u \tag{3.1.10}$$

where :

$[Q_{xxj}]_{ii}$	:	Diagonal element i of the cofactor matrix of the unknown parameters,
n	:	Total number of observations,
n_k	:	Number of observations at epoch k,
u	:	Total number of unknown parameters,
f = n - u	:	Degree of freedom for the adjustment,
m_{0j}	:	a posteriori standard deviation for the unit weight after j epochs and
m_{xji}	:	Standard deviation for the unknown parameter i after j epochs.

3.2 Ambiguity Resolution Using Statistical Criteria

The solution vector $\hat{x}_j$ contains the initial estimate for the coordinates and the real-valued ambiguities. We can generate hypothesized sets of integer ambiguities instead of the real-valued ambiguities contained in $\hat{x}_j$. Every such hypothesized set of integer ambiguities belongs to a unique set of coordinates. These coordinates can be computed in introducing a particular set of integer ambiguities as known quantities into an adjustment run. The resulting coordinates and the associated integer ambiguities are used to form a consistent alternative x_{jA} to the solution vector $\hat{x}_j$.

The question arises whether such an alternative x_{jA} can be considered compatible from the statistical point of view with the initial solution vector $\hat{x}_j$ or not.

Statistical hypothesis testing provides the tool to answer this question. The following probability statement forms the basis for such a statistical test :

$$\boxed{P\left[(x_{jA} - \hat{x}_j)^T \cdot Q_{xxj}^{-1} \cdot (x_{jA} - \hat{x}_j) \leq u \cdot m_{0j}^2 \cdot \xi_{Fu,f;1-\alpha}\right] = 1 - \alpha} \tag{3.2.1}$$

where :

$P\,[...]$	:	Probability of [...] to be true on the confidence level $1 - \alpha$,
α	:	Error probability (significance level),
$1 - \alpha$	:	Confidence level,
m_{0j}^2	:	a posteriori variance of the unit weight,
$\hat{x}_j$	:	initial solution vector,
x_{jA}	:	Hypothesized alternative for $\hat{x}_j$,
u	:	Number of elements in $\hat{x}_j$ (three coordinates plus the number of ambiguities),
$f = n - u$	:	Degree of freedom in the initial parameter estimation and
$\xi_{Fu,f;1-\alpha}$	:	range- width of the one tailed confidence range $1-\alpha$ based on Fisher's probability density function F with u and f degrees of freedom.

The probability statement (3.2.1) says that a hypothesized alternative x_{jA} must be considered compatible with $\hat{x}_j$ on the confidence level 1 - α whenever the inequality above holds.

It is known from literature e.g. [VANICEK, KRAKIWSKY, 1986] and [PELZER, 1985], that the left hand side of the inequality in equation (3.2.1) defines a u-dimensional hyperellipsoid. This hyperellipsoid can be understood as a u-dimensional confidence region centered on $\hat{x}_j$. Any alternative vector x_{jA} that falls within this hyperellipsoid must be considered compatible to the initial solution vector $\hat{x}_j$.

What has to be accomplished to resolve the ambiguities to integers is to find all consistent alternatives x_{jAi}(i = 1,...,m ; where m denotes their total number), which satisfy the probability statement (3.2.1) and select the one yielding the smallest a posteriori variance of unit weight.

Despite the fact that this ambiguity resolution approach is rigorous from a statistical point of view and in addition uniquely applicable, there are also limitations. The most serious one has to be seen in the fact, that a lot of processing power is needed to generate all the consistent alternatives x_{jAi} (coordinates and integer ambiguities) which fulfil the statement (3.2.1). Therefore we were looking for an alternative approach which is faster, but still uses the basic statistical concept presented above.

3.3 Ambiguity Resolution : Actual Implementation

The main objective of our search approach is to reduce the amount of different integer n-tuples to be introduced into subsequent adjustment runs. This is achieved by forming confidence regions for individual ambiguities and, in addition, for all possible differences between them.

We use the following information from the initial adjustment:

(i) the solution vector $\hat{x}_j$,

(ii) the corresponding cofactor matrix Q_{xxj},

(iii) the a posteriori variance factor $m_{0j}{}^2$.

The solution vector $\hat{x}_j$ is split into two subvectors $\hat{x}_C$ and $\hat{x}_N$. $\hat{x}_C$ contains the 3 coordinate parameters. $\hat{x}_N$ contains r ambiguity parameters, where r represents the total number of L1 plus L2 ambiguities.

One dimensional confidence ranges for individual ambiguities and for differences between ambiguities are formed as follows :

$$\boxed{P_i \{ x_{Ni} - \xi_{tf,1-\alpha/2} \cdot m_{xNi} \leq x_{NAi} \leq x_{Ni} + \xi_{tf,1-\alpha/2} \cdot m_{xNi} \} = 1 - \alpha} \tag{3.3.1}$$

$$\boxed{P_i \{ x_{Nik} - \xi_{tf,1-\alpha/2} \cdot m_{xNik} \leq x_{NAik} \leq x_{Nik} + \xi_{tf,1-\alpha/2} \cdot m_{xNik} \} = 1 - \alpha} \tag{3.3.2}$$

with :

$$\boxed{\begin{aligned} m_{xNi} &= m_{0j} \cdot ([Q_{xxj}]_{ii})^{1/2} \\ x_{Nik} &= x_{Ni} - x_{Nk} \;;\; i,k = 1,\ldots,r \text{ for } i \neq k \\ m_{xNik} &= m_{0j} \cdot (q_{xNik})^{1/2} \\ q_{xNik} &= [Q_{xxj}]_{ss} - 2 \cdot [Q_{xxj}]_{st} + [Q_{xxj}]_{tt};\; s = i + c \text{ and } t = k + c \end{aligned}} \tag{3.3.3}$$

where :

x_{Ni}, x_{Nk} : real-valued estimate for the ambiguity parameter i and k resp., as contained in the vector $\hat{x}_N$,

x_{NAi} : integer-valued alternative for the ambiguity parameter x_{Ni} (There are n_i different integer-valued alternatives for the ambiguity x_{Ni} as given by (3.3.1)),

x_{NAik} $:= x_{NAi} - x_{NAk}$; integer valued difference of two alternatives x_{NAi} and x_{NAk},

m_{xNi} : a posteriori standard deviation for the ambiguity parameter x_{Ni},

m_{xNik} : a posteriori standard deviation for the difference of two alternatives x_{NAi} and x_{NAk} and

$\xi_{tf,1-\alpha/2}$: upper and lower range-width of the two- tailed confidence range 1-α based on Student's probability density function t with f degrees of freedom.

The actual realization of the confidence range (3.3.1) for a particular ambiguity depends on its initial estimate x_{Ni} and the associated formal accuracy m_{xNi}. The less certain a specific ambiguity has been determined in the initial adjustment the more integer values for this ambiguity have to be searched (a total of n_i integers). All possible combinations of integer values given by these confidence ranges are used to form alternative ambiguity vectors x_{NAh}; h = 1,...,N_1 (N_1 denotes the total number of vectors), to the initial ambiguity estimates $\hat{x}_N$.

The one-dimensional search ranges given by (3.3.2) are used to decide whether a particular alternative x_{NAh} is compatible with the statistical information contained in the $\mathbf{Q}_{xxj}$ matrix. If not, this specific alternative can be neglected in further evaluation steps.

The actual search is performed as follows :

(i) In principle we perform a search over all integer alternatives x_{NAh} ; h = 1,...,N_1,. These alternatives are generated in forming all possible vector combinations using the integer values within the confidence ranges (3.3.1). There are

$$\boxed{N_1 = \prod_{i=1}^{r} (n_i)} \tag{3.3.4}$$

different vectors to be tested,

where :

n_i : denotes the number of integer values in the confidence range for ambiguity parameter i and

r : denotes the total number of L1 (and L2) ambiguities.

Despite the fact that not every vector has to be built physically in our search procedure we assume a specific ordering for the individual vectors. The very first vector x_{NA1} contains the nearest integers to each individual real-valued ambiguity. The second vector x_{NA2} is identical to the first one apart from the value for the last ambiguity x_{Nr}. It contains the second nearest integer value for x_{Nr}. The next vectors contain the third nearest value, then the fourth nearest value and so on until all integer values defined by (3.3.1) for x_{Nr} have been used. The next group of vectors starts with the second nearest integer value for x_{Nr-1} and the nearest integer values for the remaining ambiguities. Again the last ambiguity x_{Nr} is varied subsequently in the way described above to generate the next group of vectors. This procedure is repeated until all possible combinations of integer valued ambiguities as defined by (3.3.1) are formed. In summary : The last ambiguity varies fastest, the first slowest.

(ii) Let us now check whether pairs of ambiguities in the first n-tuple (n-tuple of nearest integers) x_{NA1} are in agreement with the information contained in the Q_{xxj} matrix. This is performed by testing whether the difference x_{NAik} between the first (i = 1) and the second (k = 2) ambiguity in x_{NA1} is in agreement with the criterion defined by (3.3.2).

If it is **not** in agreement, a total of

$$\boxed{N_2 = \prod_{i=3}^{r} (n_i)} \tag{3.3.5}$$

vectors can be skipped (the current vector plus the N_2 - 1 subsequent vectors; compare ordering of vectors in (i)).

If the difference x_{NAik} is in agreement with the criterion defined by (3.3.2), we test whether the differences between the first and the third and the second and the third ambiguities of n-tuple x_{NA1} are in agreement with criterion (3.3.2).

If one of these differences can not be accepted the next

$$\boxed{N_3 = \prod_{i=4}^{r} (n_i)} \tag{3.3.6}$$

vectors can be skipped.

If both differences are in agreement we continue our procedure with the differences with respect to the fourth ambiguity, then possibly to the fifth ambiguity until we eventually arrive at the last ambiguity in x_{NA1}.

(iii) Step (ii) has to be repeated for the next valid vector until all alternatives x_{NAh} are analyzed. The number of the next vector to be tested can be computed as the number of the last vector considered in (ii) plus the number of n-tuples to be skipped.

The result of this search is a list of accepted integer ambiguity vectors x_{NAa} with a = 1,...,s where s denotes the total number of accepted ambiguity vectors. Each of these accepted vectors is introduced into a subsequent adjustment run. The integer ambiguities are treated in these adjustments as known quantities. The resulting solution vectors $\hat{x}_{CAa}$ (coordinates only) and the standard deviations m_{0a} are used to assess the quality of results. The integer vector yielding the smallest standard deviation is selected for the final solution unless:

(i) the corresponding position vector $\hat{x}_{CA}$ is not statistically compatible with the initial estimate $\hat{x}_C$, or

(ii) its standard deviation is not compatible with σ_0, or

(iii) there are several vectors yielding "almost" identical standard deviations.

Conditions (i) to (iii) can be checked by means of statistical hypothesis testing. The employed statistic to test condition (ii) is the so called model test or the Ψ^2 test of the variance factor (Vanicek, Krakiwsky, 1986). The null hypothesis H_0 for this test reads:

$$H_0 : m_{0s}^2 = \sigma_0^2$$

where m_{0s} denotes the smallest standard deviation.

The alternative hypothesis H_1 reads :

$$H_1 : m_{0s}^2 \neq \sigma_0^2 \tag{3.3.8}$$

and the corresponding test statistic :

$$T_s = \frac{m_{0s}^2}{\sigma_0^2} \tag{3.3.9}$$

Under the assumption that the null hypothesis is valid the probability density of T_s is $\Psi^2{}_{f/f}$ (chi-squared over f with f degrees of freedom). m_{0s} and σ_0 can be considered compatible whenever :

$$\xi_{\Psi 2f/f,a/2} \leq T_s \leq \xi_{\Psi 2f/f,1-a/2} \tag{3.3.10}$$

where :

$\xi_{\Psi 2f/f,\alpha/2}$: lower boundary of the 1-α confidence interval for T_s and

$\xi_{\Psi 2f/f,1-\alpha/2}$: upper boundary of the 1-α confidence interval for T_s.

Assuming that the estimates for the smallest standard deviation m_{0s} and the second best standard deviation $m_{0s'}$ are independent, the following statistical test can be employed to check condition (iii). The null hypothesis H_0 for this test reads :

$$H_0 \;:\; m_{0s}^2 \;=\; m_{0s'}^2 \tag{3.3.11}$$

The alternative hypothesis H_1 reads :

$$H_1 \;:\; m_{0s}^2 \;\neq\; m_{0s'}^2 \tag{3.3.12}$$

and the corresponding test statistic :

$$T_{s'} \;=\; \frac{m_{0s}^2}{m_{0s'}^2} \tag{3.3.13}$$

Under the assumption that the null hypothesis is valid the probability density of $T_{s'}$ is $F_{f1,f2}$ (F density with f_1 and f_2 degrees of freedom). m_{0s} and $m_{0s'}$ can be considered compatible whenever:

$$T_{s'} \;\leq\; \xi_{Ff1,f2;1-\alpha/2} \tag{3.3.14}$$

where :

$\xi_{Ff1,f2;1-\alpha/2}$: boundary of the 1-α confidence interval for $T_{s'}$ and

$f = f_1 = f_2$: degree of freedom in the estimation of m_{0s} and $m_{0s'}$.

If the statement (3.3.14) above is true, more than one ambiguity vector would have to be considered as valid candidates for the final solution, which means that the attempt to resolve ambiguities to integers can not be terminated successfully with the available data set.

Whenever one of the statements (i),(ii) or (iii) above is true, the available data set (measurements) does not suffice to reliably fixing the ambiguities to integer values. Additional measurements are needed to cure this deficiency.

3.4 Additional Properties of x_{Nik}

Let us first concentrate on the case where x_{Nik} represents the difference between ambiguities referring to the same carrier (either L_1 or L_2).

Using double-differenced phase observables leads to solving for differences between ambiguities (of satellite pairs) to avoid rank deficiencies in the normal equation system. The selection of a reference satellite is more or less arbitrary. However, the ability to resolve ambiguities may depend on this selection. Therefore, the ambiguities which are theoretically best determined should be selected as unknowns (G. BLEWITT :"An optimal double-differencing transformation", 1989). It is not necessary in our approach to pay attention to this point, since, as explained in the previous section, we consider criterion (3.3.2) for each possible alternative selection of the ambiguities. The proposed search technique considers all the possible satellite combinations and thus does not depend on the reference satellite.

If ambiguity differences belonging to the same satellites but different frequencies are formed, the x_{Nik} represents the so called wide-lane ambiguity (HATCH, 1985).

3.5 An optimal dual frequency search

As soon as there are L_1 **and** L_2 measurements available the search technique above can be improved considerably with an additional test :

Equation (3.1.1) denotes the observation equation either for an L_1 or an L_2 observable. Considering short antenna separation distances (highly correlated atmosphere) the difference between simultaneous L_1 and L_2 measurements (same pair of satellites) can be expressed by:

$$\lambda_1 \cdot N_1^{nm} - \lambda_2 \cdot N_2^{nm} = dd(l_1) - dd(l_2) + v_1 - v_2$$

(3.5.1)

Equation (3.5.1) may be interpreted as an observation equation for one unknown parameter p :

$$p = N_1^{nm} - \frac{\lambda_2}{\lambda_1} \cdot N_2^{nm}$$

(3.5.2)

All observation equations (3.5.1) to the same pair of satellites might be combined in a least squares sense to determine the optimum estimate for p and its standard deviation m_p.

Now,

$$\hat{p} = N_1^{nm} - \frac{\lambda_2}{\lambda_1} \cdot N_2^{nm}$$

(3.5.3)

represents a straight line in a two-dimensional parameter (sub-) space defined by the parameters N_1^{nm} and N_2^{nm} (see Figure 3.1). The associated standard deviation m_p can be used to define a confidence region. m_p is very small (typically a few millimeters) since equations (3.5.1) do not contain geometry and clock related information.

It is **not** necessary to actually form equations (3.5.1) explicitly with additional observation equations because this information is already contained in the solution vector $\hat{x}_j$. If we form the linear combination :

$$x_{Lik} = x_{Ni} - \frac{\lambda_2}{\lambda_1} \cdot x_{Nk}$$

(3.5.4)

using the ambiguities x_{Ni} and x_{Nk} referring to the same pair of satellites but to the L_1 and L_2 carriers, we see that the associated standard deviation m_{xLik} is very small (as a matter of fact it is identical to m_p).

Therefore, we introduce in addition to criteria (3.3.1) and (3.3.2) a new criterion which is used, if, and only if x_{Ni} and x_{Nk} represent ambiguities for the same pair of satellites but for different carriers (L_1 and L_2).

$$\boxed{P_i \{ x_{Lik} - \xi_{tf,1-a/2} \cdot m_{xLik} \leq x_{LAik} \leq x_{Lik} + \xi_{tf,1-a/2} \cdot m_{xLik} \} = 1 - \alpha} \tag{3.5.5}$$

where :

$$\boxed{x_{LAik} = x_{NAi} - \frac{\lambda_2}{\lambda_1} \cdot x_{NAk}} \tag{3.5.6}$$

Whenever an alternative pair of integer ambiguities x_{NAi} and x_{NAk} (same pair of satellites, different frequencies) is introduced into equation (3.5.4) and the resulting value x_{LAik} is not located within the confidence range defined by (3.5.5) all n-tuples x_{NAh} containing this ambiguity pair can be skipped.

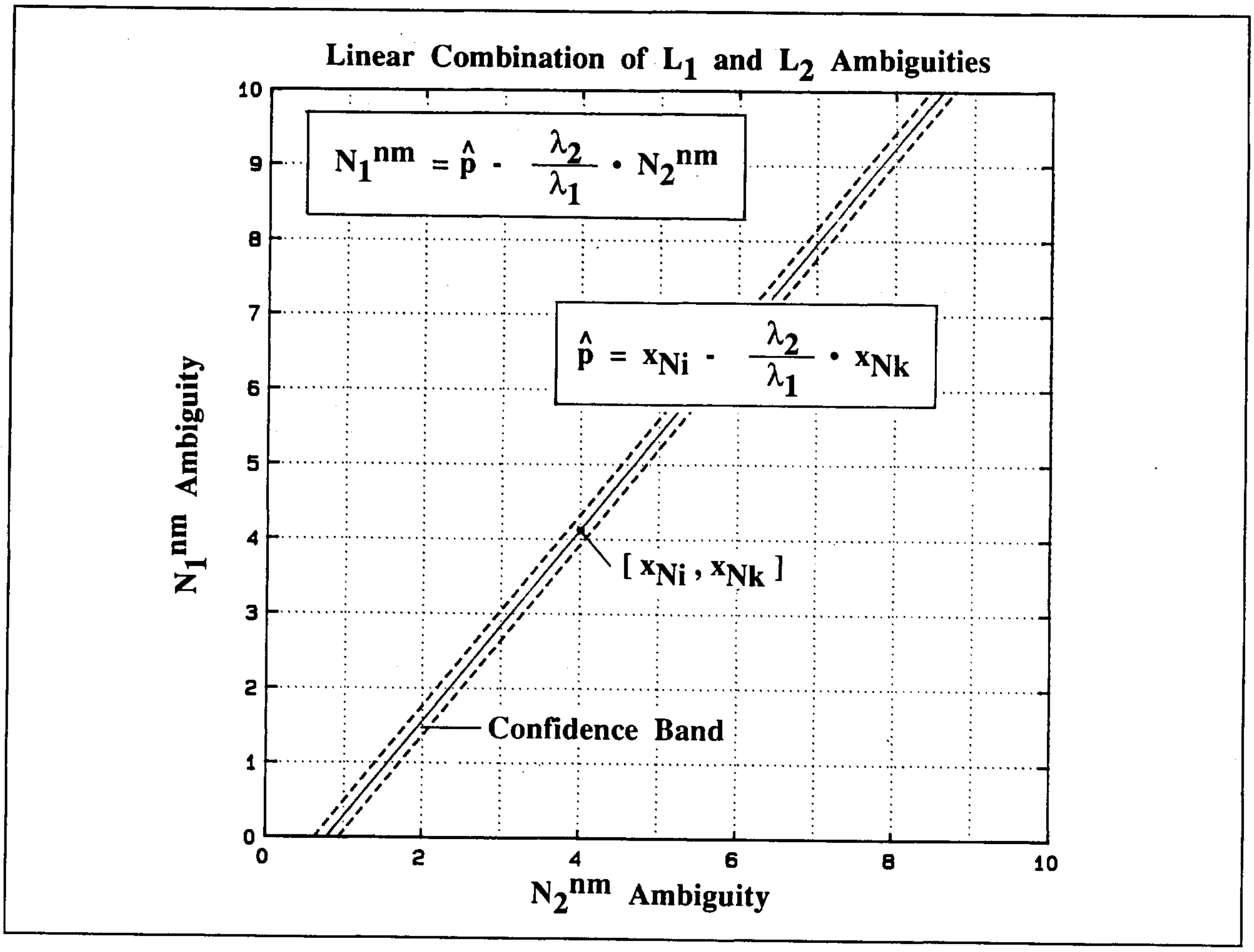


Fig. 3.1 : *Graphical representation of the linear combination between L1 and L2 ambiguities.*

III : 4. Computations with real data

The data sets used to demonstrate a few properties of our proposed fast ambiguity resolution approach are from the Turtmann 89 campaign. A detailed description of the Turtmann test range can be found in [ROTHACHER et al., 1986]. The results of this 89 campaign are fully documented in [ROTHACHER et al., 1990]. Measurements with WM102 receivers on a baseline from TURT to ERGI collected on July, 7th have been selected to demonstrate the search technique. The slope distance between the two points is two kilometers and the height difference is about 500 meters.

Four different examples will be presented. The underlying measurement scenarios differ in the number of measurements used, in the way the measurements are spaced in time and in the availability of single or dual frequency data.

4.1 Example 1

Processing Scenario : Processing of five minutes of L_1 and L_2 observations (six epochs) using one minute data sampling for five satellites. A total of 47 double-differenced observations have been processed (one L_2 measurement was missing).

Results of the initial adjustment : The a posteriori estimate of the standard deviation of the unit weight is 3 mm. The standard deviations for the real-valued ambiguity estimates are in the order of two to eight tenths of a cycle. A straight fixing to the nearest integer values would yield incorrect results as the values in the column "truth" in Table 4.1. show. This "truth" has been determined in processing the entire set of measurement data available for this baseline. The standard deviations in Table 4.1 together with the real-valued ambiguity estimates are used subsequently to form the confidence ranges as defined in (3.3.1).

The coordinates and the slope distance estimated prior to ambiguity resolution (Table 4.2) show that the initial position estimate is not in agreement with the true position in the order of half a meter.

Table 4.1.: Ambiguity estimates and standard deviations :

No	Freq.	Sat.	Ref. Sat.	Ambiguity	Standard Deviations	Truth
				[cycles]	[cycles]	[cycles]
1	1	6	12	-9.07	0.78	-8
2	1	9	12	-2.74	0.31	-3
3	1	8	12	-10.26	0.83	-9
4	1	11	12	-17.53	0.44	-18
5	2	6	12	-6.84	0.61	-6
6	2	9	12	-1.83	0.24	-2
7	2	8	12	-6.98	0.65	-6
8	2	11	12	-13.69	0.34	-14

Table 4.2 : Resulting coordinates and slope distance prior to ambiguity resolution :

Quantity	Estimates	Truth	Difference
	[m]	[m]	[m]
X - Coordinate	4375518.973	4375519.764	0.791
Y - Coordinate	593010.606	593010.699	0.093
Z - Coordinate	4588793.269	4588793.382	0.113
Slope distance	2005.187	2005.675	0.488

Integer Ambiguity Search : Based on the results provided by the initial adjustment the search is performed as described in section 3.3.

Table 4.3 : Differences between ambiguities and the associated standard deviations

Amb 1	Amb 2	Difference	Standard Deviations	Truth	Comment
		[cycles]	[cycles]	[cycles]	
1	2	-6.33	0.52	-5	
1	3	1.19	0.06	1	
1	4	8.46	0.53	10	
1	5	-2.24	0.17	-2	wide lane
1	6	-7.24	0.58	-6	
1	7	-2.10	0.13	-2	
1	8	4.62	0.56	6	
2	3	7.52	0.58	6	
2	4	14.79	0.23	15	
2	5	4.09	0.36	3	
2	6	-0.92	0.07	-1	wide lane
2	7	4.23	0.41	3	
2	8	10.95	0.18	11	
3	4	7.27	0.58	9	
3	5	-3.43	0.23	-3	
3	6	-8.43	0.63	-7	
3	7	-3.28	0.18	-3	wide lane
3	8	-3.43	0.62	5	
4	5	-10.69	0.39	-12	
4	6	-15.70	0.26	-16	
4	7	-10.55	0.43	-12	
4	8	-3.84	0.10	-4	wide lane
5	6	-5.01	0.41	-4	
5	7	0.14	0.05	0	
5	8	6.86	0.41	8	
6	7	5.15	0.45	4	
6	8	11.86	0.18	12	
7	8	6.71	0.45	8	

Table 4.3 shows the differences between real-valued ambiguity estimates and the associated standard deviations. These figures are used in turn to form the one-dimensional confidence ranges as defined by (3.3.2). The factor to scale the confidence intervals has been set to

$\xi_{tf,1-\alpha/2}$	=	3.6
f	=	36
α	=	0.1 %

The differences labeled "wide-lane" in the comment column are the differences between L_1 and L_2 ambiguities to identical satellite pairs. The associated standard deviations are below two tenths of a wide lane cycle.

Due to the fact that there are L_2 measurements available the additional test described in section 3.5 can be employed for the search. Table 4.4 shows the resulting values for x_{Lik} and the corresponding standard deviations m_{xLik}. Note that these linear combinations are known on an accuracy level of a few millimeters.

Table 4.4 : *Linear combinations of L_1 and L_2 ambiguities*

L1	L2	x_{Lik}	m_{xLik}	Truth
		[cycles]	[cycles]	[cycles]
1	5	-0.30	0.01	-0.30
2	6	-0.40	0.01	-0.43
3	7	-1.31	0.01	-1.30
4	8	-0.05	0.01	-0.03

According to the formula (3.3.4) there are more than 24,000 different ambiguity pairs to be considered as alternatives to the initial solution vector $\hat{x}_j$. Out of these 24,000 alternatives only two pairs have been accepted by the search procedure described in chapter 3.3. These two n-tuples have been individually analyzed in an adjustment run where the integer ambiguities have been introduced as known quantities. Table 4.5 shows the a posteriori standard deviations m_{0a} and the test statistic $T_{s'}$ (see 3.3) for the two analyzed n-tuples.

Table 4.5 : *Resulting standard deviations m_{0a} and test statistic T_s'*

Vector	Standard Deviations	$T_{s'}$	$\xi_{Ff,f;1-\alpha/2}$ $\alpha = 5$ %	Test Decision
	[mm]			
1	4	-	-	-
2	14	12.2	1.8	Failed

According to the test statistic Ts' there is a significant difference between the standard deviation for the first vector and the second vector. Therefore vector one is taken for the final solution. The results obtained using this vector are shown in Table 4.6.

Table 4.6 : *Resulting coordinates and slope distance using ambiguity n-tuple 1*

Quantity	Estimates	Truth	Difference
	[m]	[m]	[m]
X - Coordinate	4375519.764	4375519.764	-0.000
Y - Coordinate	593010.699	593010.699	0.000
Z - Coordinate	4588793.361	4588793.382	-0.021
Slope distance	2005.682	2005.675	0.007

4.2 Example 2

Processing Scenario : Processing of five minutes of L_1 observations (six epochs) using one minute data sampling for five satellites. A total of 24 double-differenced observations have been processed.

Results of the Initial Adjustment : The a posteriori estimate for the standard deviation of the unit weight is 3.2 mm.

Integer Ambiguity Search : The factor to scale the confidence intervals has been set to

$\xi_{tf,1-\alpha/2}$	=	3.7
f	=	17
α	=	0.1 %

For this L1 measurement scenario there would have been 1,440 different ambiguity vectors to be processed using a general search technique. Our approach has selected 160 vectors which had to be analyzed in subsequent adjustment runs. Table 4.8 shows the resulting standard deviations a posteriori for the five vectors yielding the smallest standard deviations.

Even if the first ambiguity vector yields an acceptable standard deviation compared to σ_0, it can not be taken as the final solution, because there are several other vectors yielding almost identical standard deviations. This demonstrates the fact that the available L_1 measurements after these five minutes of observations do not suffice to resolve the ambiguities to integer values. As the example 1 above has shown, adding the L_2 measurements for this measuring period allows for resolving the ambiguities without any difficulties.

Table 4.7.: *Ambiguity estimates and standard deviations*

No	Freq.	Sat.	Ref. Sat.	Ambiguity	Standard Deviations	Truth
				[cycles]	[cycles]	[cycles]
1	1	6	12	-8.71	1.15	-8
2	1	9	12	-2.69	0.47	-3
3	1	8	12	-9.86	1.22	-9
4	1	11	12	-17.31	0.65	-18

Table 4.8 : *Resulting standard deviations m_{0a} and test statistic T_s'*

Vector	Standard Deviations	$T_{s'}$	$\xi_{Ff,f;1-\alpha/2}$ $\alpha = 5\ \%$	Test Decision
	[mm]			
1	3.0	-	-	-
2	3.1	1.07	2.30	Accepted
3	3.1	1.07	2.30	Accepted
4	3.5	1.36	2.30	Accepted
5	3.7	1.52	2.30	Accepted

4.3 Example 3

Processing Scenario : Processing of two blocks of five minutes of L_1 observations using one minute data sampling for five satellites. The two measurement blocks are separated by one hour. A total of 48 double-differenced observations are processed.

Results of the Initial Adjustment : The a posteriori estimate of the standard deviation for the unit weight m_{0j} is 3.2 mm.

Integer Ambiguity Search : The factor to scale the confidence intervals has been set to

$\xi_{tf,1-\alpha/2}$	=	3.3
f	=	37
α	=	0.1 %

For this measurement scenario there would have been 63,500 different ambiguity vectors to be processed using a general search technique. Our search approach has selected 10,000 vectors which have been analyzed in subsequent adjustment runs. Table 4.10 shows the resulting a posteriori standard deviations for the first five vectors.

Table 4.9 : *Ambiguity estimates and standard deviations*

No	Freq.	Sat.	Ref. Sat.	Ambiguity	Standard Deviations	Truth
				[cycles]	[cycles]	[cycles]
1	1	6	12	-8.02	0.88	-8
2	1	9	12	-2.69	0.40	-3
3	1	8	12	-9.06	0.93	-9
4	1	11	12	-17.59	0.52	-18
5	1	6	12	-6.86	0.36	-7
6	1	9	12	-2.49	0.84	-2
7	1	11	12	-11.86	0.42	-12
8	1	13	12	-20.58	0.37	-21

Table 4.10 : *Resulting standard deviations m_{0a} and test statistic $T_{s'}$*

Vector	Standard Deviations	$T_{s'}$	$\xi_{Ff,f;1-\alpha/2}$ $\alpha = 5\ \%$	Test Decision
	[mm]			
1	6.0	-	-	--
2	8.7	2.10	1.80	Failed
3	11.5	3.67	1.80	Failed
4	11.6	3.74	1.80	Failed
5	12.1	4.07	1.80	Failed

Ambiguity vector one can be accepted for the final solution because the vector yielding the second best standard deviation did not pass the statistical test. The corresponding results are shown in Table 4.11.

Table 4.11 : Resulting coordinates and slope distance

Quantity	Estimates	Truth	Difference
	[m]	[m]	[m]
X - Coordinate	4375519.727	4375519.764	-0.037
Y - Coordinate	593010.692	593010.699	-0.007
Z - Coordinate	4588793.351	4588793.382	-0.031
Slope distance	2005.658	2005.675	-0.017

4.4 Example 4

Processing Scenario : Processing one minute of L_1 and L_2 observations (two epochs) using one minute data sampling for five satellites. A total of 16 double-differenced observations have been processed.

Results of the Initial Adjustment : The a posteriori estimate of standard deviation for the unit weight is 2 mm.

The standard deviations in Table 4.12 are in the order of one to three cycles. An attempt to determine the correct n-tuple of ambiguities is undertaken despite the fact that the standard deviations for the ambiguities do not look very promising.
The coordinates and the slope distance estimated prior to ambiguity resolution are listed in Table 4.13.

Table 4.12 : Ambiguity estimates and standard deviations

No	Freq.	Sat.	Ref. Sat.	Ambiguity	Standard Deviations	Truth
				[cycles]	[cycles]	[cycles]
1	1	6	12	-5.05	2.83	-9
2	1	9	12	-1.40	1.13	-3
3	1	8	12	-4.92	3.02	-9
4	1	11	12	-13.64	1.57	-18
5	2	6	12	-3.92	2.20	-7
6	2	9	12	-0.78	0.88	-2
7	2	8	12	-2.82	2.36	-6
8	2	11	12	-10.67	1.22	-14

Table 4.13 : Resulting coordinates and slope distance

Quantity	Estimates	Truth	Difference
	[m]	[m]	[m]
X - Coordinate	4375519.959	4375519.764	0.195
Y - Coordinate	593010.228	593010.699	-0.471
Z - Coordinate	4588793.523	4588793.382	0.141
Slope distance	2005.383	2005.675	0.292

Integer Ambiguity Search : Based on the results provided by the initial adjustment the search is performed as described in chapter 3.3. The factor to scale the confidence intervals has been set to

$\xi_{tf,1-\alpha/2}$	=	6.9
f	=	5
α	=	0.1 %

Due to the fact that there are L_2 measurements available the additional test described in section 3.5 can be employed for the search. Table 4.14 shows the resulting values for x_{Lik} and the corresponding standard deviations m_{xLik}. Note that these linear combinations are known to a level of accuracy of a few millimeters.

Table 4.14 : *Linear combinations of L_1 and L_2 ambiguities*

L1	L2	x_{Lik}	m_{xLik}	Truth
		[cycles]	[cycles]	[cycles]
1	5	-0.02	0.01	-0.02
2	6	-0.40	0.01	-0.43
3	7	-1.30	0.01	-1.30
4	8	0.05	0.01	-0.03

According to the formula (3.3.4) there had to be more than $1.17 \cdot 10^{11}$ different ambiguity vectors to be considered in a general search. Only 14 vectors have been accepted by the search procedure described in chapter 3.3. These 14 vectors have been individually analyzed in an adjustment where the integer ambiguities have been introduced as known quantities.

Table 4.15. shows the a posteriori standard deviations m_{0a} and the test statistic $T_{s'}$ (see 3.3) for the five smallest standard deviations.

Table 4.15 : Resulting standard deviations m_{0a} and test statistic $T_{s'}$

Vector	Standard Deviations	$T_{s'}$	$\xi_{Ff,f;1-\alpha/2}$ $\alpha = 5$ %	Test Decision
	[mm]			
1	4.0	-	-	----
2	10.0	6.2	2.6	Failed
3	21.1	9.1	2.6	Failed
4	21.6	29.2	2.6	Failed
5	26.3	42.0	2.6	Failed

Vector number one can be accepted for the final solution because vector number two is significantly different. The results obtained using n-tuple number one are shown in Table 4.16.

Table 4.16 : Resulting coordinates and slope distance

Quantity	Estimates	Truth	Difference
	[m]	[m]	[m]
X - Coordinate	4375519.760	4375519.764	0.004
Y - Coordinate	593010.700	593010.699	-0.001
Z - Coordinate	4588793.363	4588793.382	0.019
Slope distance	2005.679	2005.675	-0.004

III : 5. Summary and Conclusions

We have shown that using the "Fast Ambiguity Resolution Approach (FARA)" enables us to resolve ambiguities with one minute of L_1 and L_2 measurements when observing five satellites on a short baseline (2 km). In addition we have shown that it takes considerably longer to resolve ambiguities if only L_1 data is available. The "Fast Ambiguity Resolution Approach" can be characterized by the following, so far unique, properties, namely it :

(i) is generally applicable (for L_1 measurements only or for dual band measurements)

(ii) is rigorous from a statistical standpoint,

(iii) does not depend on a particular selection of the reference satellite,

(iv) is capable to decide whether the available information (measurements) are sufficient to correctly resolve the ambiguities or not.

The technique is based on the information provided by an initial adjustment, namely the solution vector (coordinates and real-valued ambiguity estimates) and the corresponding variance - covariance matrix.

The limited testing which has been performed using data from classic static GPS campaigns observing a five satellite constellation has led to the following conclusions :

(i) It can be expected that the ability to resolve ambiguities will improve considerably in terms of the required site occupation time as soon as six and more satellites will be available.

(ii) One minute of L_1 and L_2 observations might not be the shortest site occupation time which allows for a successful ambiguity resolution. Test data with a higher sampling rate will have to be gathered and analyzed to determine the limits of the "Fast Ambiguity Resolution Approach".

A field campaign has been conducted in spring 1990 addressing questions like shortest possible site occupation time and possible consequences to the practical utilization of this technique. The results and findings will be discussed in the next part of this work.

Rapid static positioning based on the "Fast Ambiguity Resolution Approach" is going to change detail surveying. Individual site occupations of one to two minutes will suffice to resolve ambiguities for phase measurements taken with dual band receivers. The accuracies to be achieved with this technique will be on the centimeter level or even better.

Part IV : Measurements, Computations and Results

Rapid Differential Positioning based on the Fast Ambiguity Resolution Approach : Extensive Tests [1)]

1) Parts of this chapter have been published in the Proceedings of the Second International Symposium on Precise Positioning with the Global Positioning System "GPS'90", Ottawa, Canada, Sept. 3 - 7, 1990

IV : 1. Introduction

1.1 Objectives

Processing selected measurements from the "Turtmann 89" campaign using the "Fast Ambiguity Resolution Approach" (FARA) has shown that one to two minutes worth of L_1 and L_2 data are sufficient to resolve initial phase ambiguities. The question was raised how this approach performs e.g. in terms of the required site occupation time, the number of measurements, the satellite constellation and the required computing time. In order to answer these questions the "FARA 90" campaign has been conducted. The main objective of this four day campaign was to gather measurements to evaluate the "FARA" under different processing scenarios. Each individual processing scenario has been designed to answer one specific question. The corresponding questions are :

(i) How well do classical ambiguity resolution approaches perform compared to the "FARA"?.

(ii) Are five minutes worth of L_1 and L_2 data in any case sufficient to resolve the ambiguities and what kind of accuracy can be obtained?

(iii) Might one minute worth of L_1 and L_2 data still suffice to resolve ambiguities? If yes, is there a significant change in accuracy compared to the five minute case?

(iv) What is the performance if only L_1 measurements would be used?

(v) How many measurements are required if the same site will be visited twice or more times and what kind of accuracy can be expected?

(vi) Is it mandatory to observe at least five satellites simultaneously or will four satellites be enough?

(vii) What is the smallest amount of measurements which will still guarantee a successful ambiguity resolution?

(viii) Where is the limit in terms of antenna separation distance?

(ix) Is an instantaneous ambiguity resolution possible or not?

Answering these questions will provide the necessary information to evaluate the potential of the "FARA" as a tool to resolve the initial phase ambiguities for accurate positioning techniques with GPS .

1.2 The Implementation of "FARA"

A first version of the "FARA" as described in part III has been implemented in the Bernese GPS Software Package Version 3.2 (ROTHACHER et al., 1990). All tests and results presented in this chapter have been performed with this software package either running on a mainframe system or on a PC.

IV : 2. The "FARA" Campaign 90

2.1 Objectives

Since the purpose of the "FARA 90" campaign was to provide measurements to evaluate and validate the "FARA", the following test design has been chosen : The test range had to be selected with regard to the availability of an accurate ground truth and to its accessibility. Because of the expected limitations of the "FARA" due to unmodelled ionospheric effects, the tests were restricted to a distance range of a few meters up to 15 kilometers. Even if the accuracies with the short observation periods are of interest, more attention was paid to the mechanisms and the performance of ambiguity resolution. The general rule in planning this campaign was to be on the safe side concerning the number of measurements as well as the number of satellites observed. The Bernese GPS Software Package provides the necessary tools to analyze subsets of the originally gathered measurements. This enables to create and process a variety of different measuring scenarios without taking any risk as to gather too few data in the field for a successful data analysis.

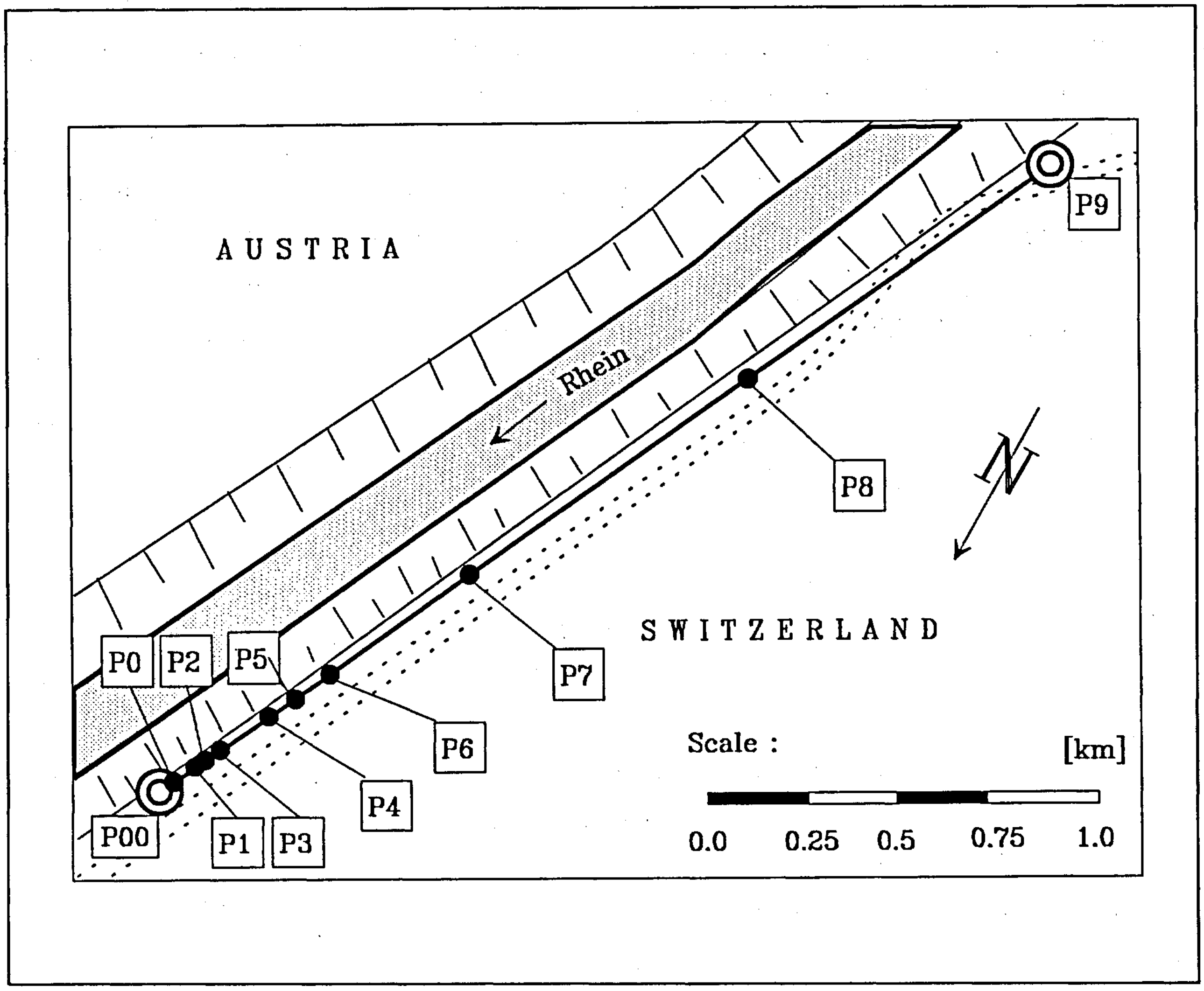


Figure 2.2.1 : Sketch of the Heerbrugg Test Range

2.2 The Test Ranges

Two different test ranges have been selected for the "FARA 90" campaign. The first test range is an EDM test range of Leica Heerbrugg Ltd (Switzerland). It is located on the left border of the river Rhein right next to the border to Austria. This test range includes 11 pillars with separation distances from eight meters up to three kilometers (see Figure 2.2.1).

The inter-pillar distances as well as the height differences are known very accurately: The accuracies of the slope distances are in the order of a millimeter for the longer distances and in the sub-millimeter level for shorter distances. The accuracies of the levelled height differences are better than a millimeter. Figure 2.2.2 shows the "ground truth" for the slope distances as well as the levelled height differences. The following simplified model was used :

Due to the fact that the levelled height differences can not be compared directly with the height differences resulting from a GPS position adjustment in the satellite datum (WGS - 84), the levelled height differences had to be transformed into the GPS datum. A simplified model has been used to transform the levelled height differences into the GPS datum (WGS - 84). The model is:

$$\boxed{\Delta h_{(WGS)} = \Delta h + \Delta N + \Delta_{ell}} \tag{2.2.3}$$

where :

Δh	:	denotes the levelled height differences,
ΔN	:	denotes the corrections to transform levelled heights to ellipsoidal heights based on the Swiss Geodetic datum, and
Δ_{ell}	:	denotes the correction to be applied to transform the ellipsoidal height differences from the Swiss Geodetic Datum to the GPS datum.

Table 2.2.4 shows the applied corrections and the final height differences which will be used in later comparisons.

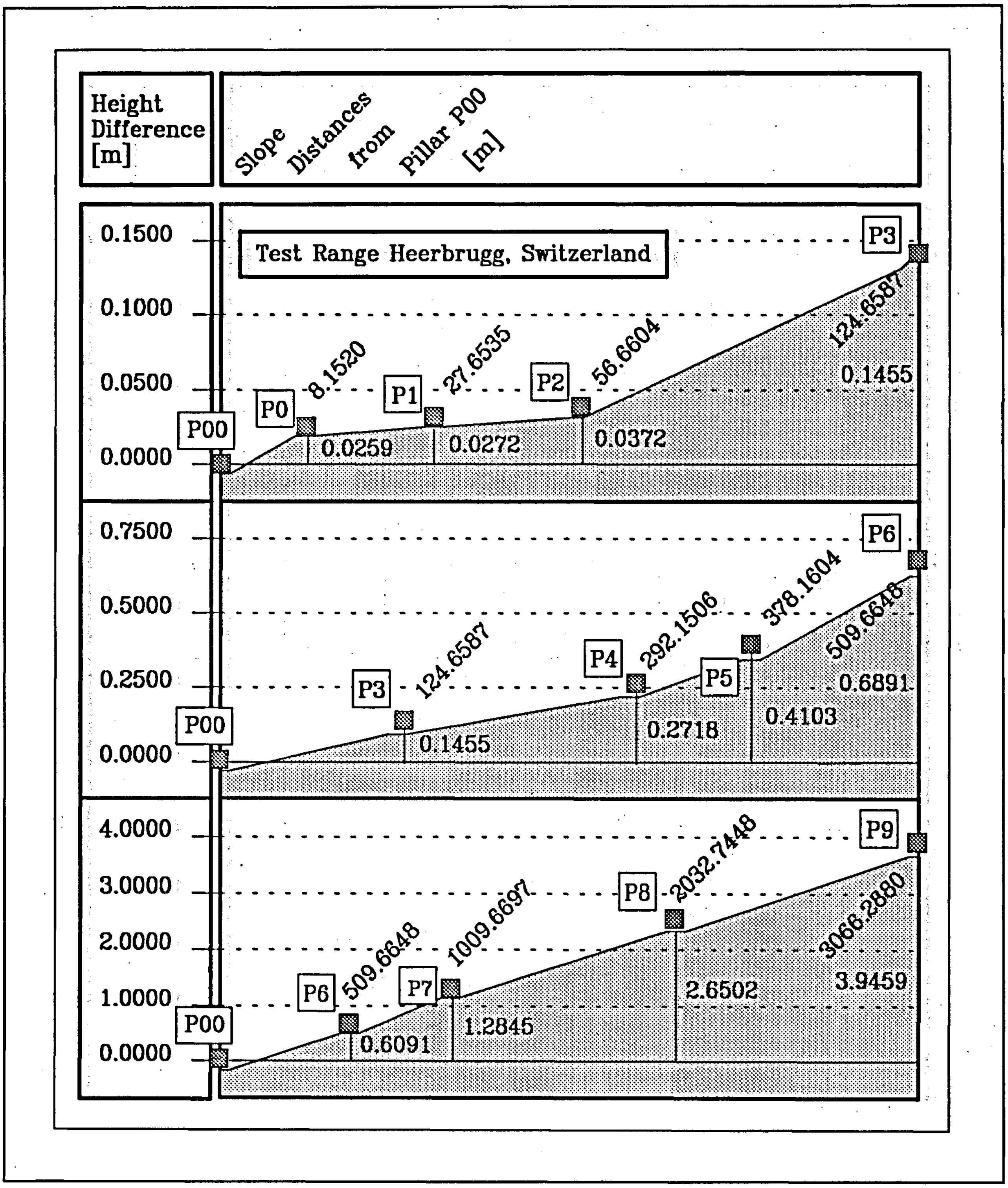


Figure 2.2.2 : The "Ground Truth" for the EDM Test Range Heerbrugg

Table 2.2.4 : Transformation of Height Differences into the GPS Datum

Line	Δh levelled	Slope Distance	ΔN	$\Delta h_{(1903)}$	Δ_{ell}	$\Delta h_{(WGS)}$
	[m]	[m]	[mm]	[m]	[mm]	[m]
P00 - P0	0.0259	8	0	0.026	0	0.026
P00 - P1	0.0272	27	0	0.027	1	0.028
P00 - P2	0.0372	56	0	0.037	2	0.039
P00 - P3	0.1455	124	0	0.146	3	0.149
P00 - P4	0.2718	292	1	0.273	8	0.281
P00 - P5	0.4103	378	1	0.411	10	0.421
P00 - P6	0.6891	509	2	0.691	14	0.705
P00 - P7	1.2845	1009	4	1.289	28	1.317
P00 - P8	2.6502	2032	8	2.658	56	2.714
P00 - P9	3.9459	3066	12	3.958	85	4.043

where :

$\Delta h_{(1903)}$: height difference in the Swiss geodetic datum (CH - 1903),
$\Delta h_{(WGS)}$: height difference as to be observed in the GPS datum (WGS - 84).

Although the levelled height differences are accurate to a tenth of a millimeter, the accuracies of the transformed height differences are assumed not to be better than a few millimeters.

The second test range consists of five points with antenna separation distances of five, seven, ten and twelve kilometers. These points were selected "ad hoc" to test the limits of "FARA" in terms of the antenna separation distance. There is however no "ground truth" available for these baselines.

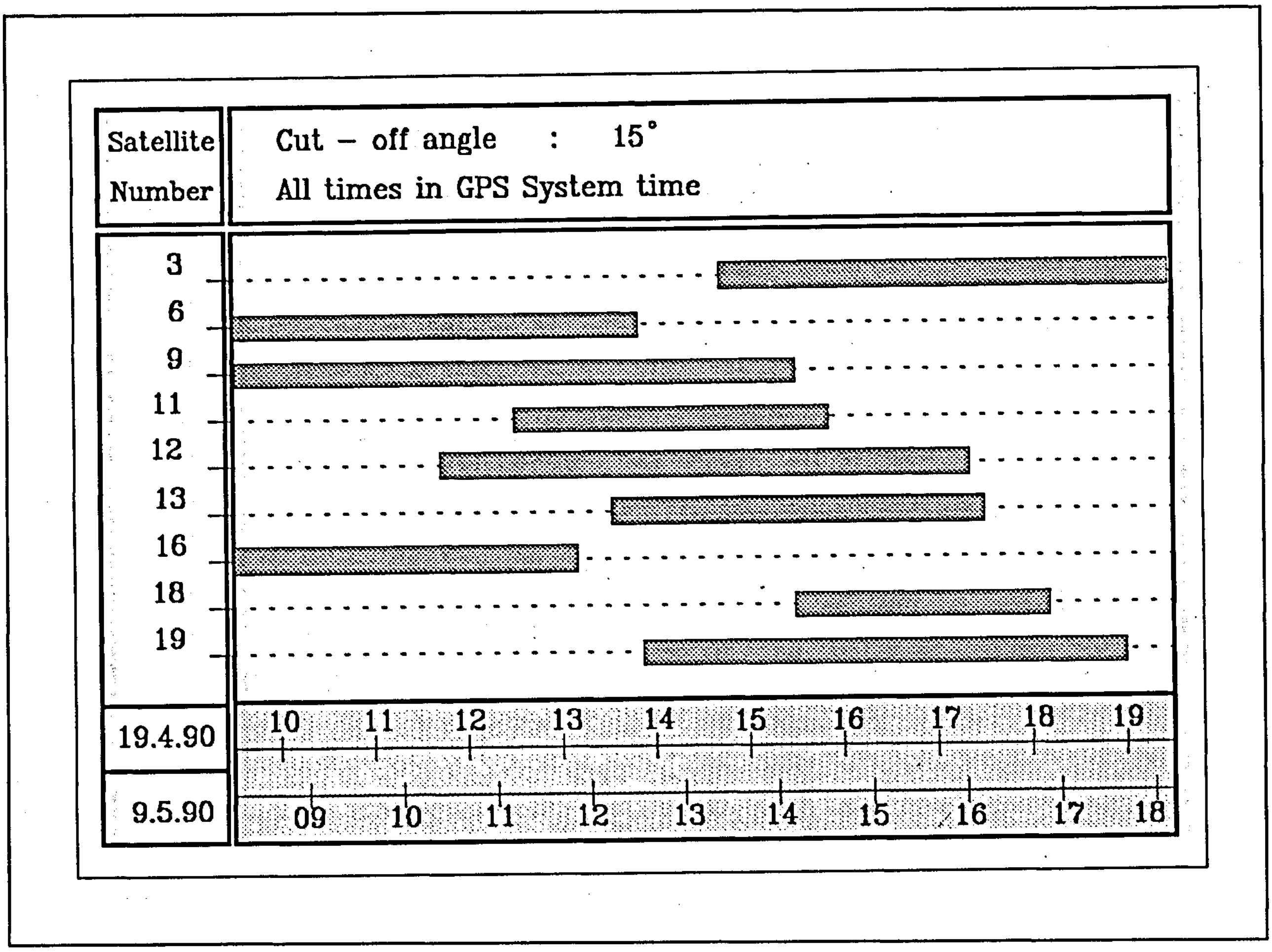


Figure 2.3.1 : Satellite Visibility for the 19th of April 1990 and the 9th of May 1990

2.3 The Satellite Constellation

The "FARA 90" campaign has been conducted on the Heerbrugg Test Range on four days, namely on the 19th and 20th of April with WM102 receivers and on the 8th and 9th of May with Trimble 4000 SLD receivers. The time windows were selected to allow to track at least four satellites simultaneously. The satellite constellation available over Switzerland at that time provided four and more satellites for an observation period of more than five hours (see Figures 2.3.1 and 2.3.2). There was even a one hour period with a six satellite constellation available. The measurements on the second test range have been taken on the 28th of June 1990.

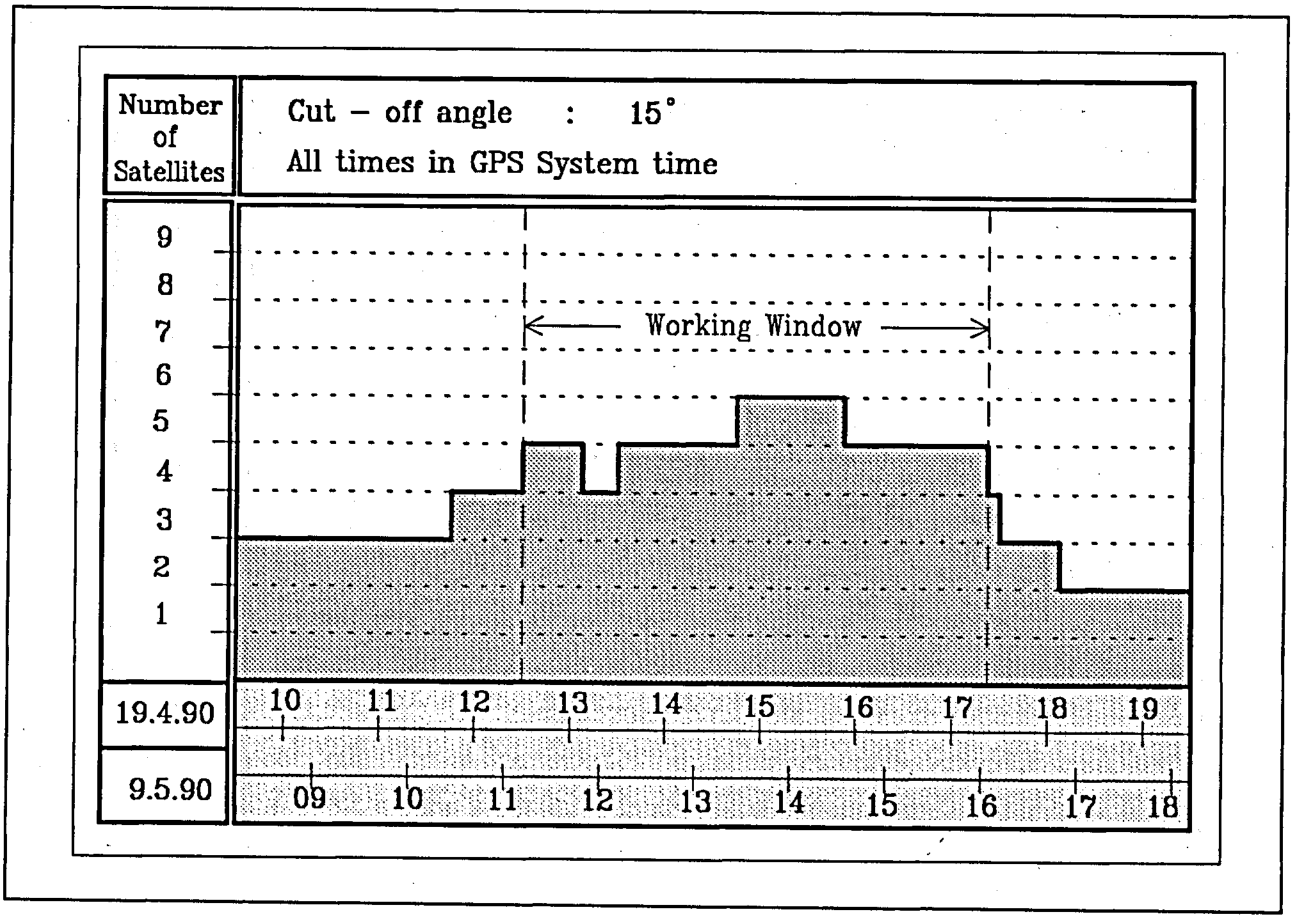


Figure 2.3.2 : Satellite Availability for the 19th of April 1990 and the 9th of May 1990

Figures 2.3.1 and 2.3.2 show the satellite constellations on the 19th of April and on the 9th of May. Due to the fact that the satellite constellations are repeated every day about four minutes earlier than the day before, the constellation for the 19th of April is identical with the one on the 9th of May with the only difference that the latter shows up at about one hour and twenty minutes earlier than the one on the 19th of April.

Figure 2.3.3 shows the GDOP (Geometrical Dilution of Precision) and PDOP (Positional Dilution of Precision) values for the selected working window (see Part V for a brief introduction to the DOP- values). These DOP's will be used as an integral indicator to assess the quality of a specific constellation as far as geometrical aspects are concerned. As can be seen in Figure 2.3.3 the values for GDOP and PDOP are strongly correlated with the number of available satellites.

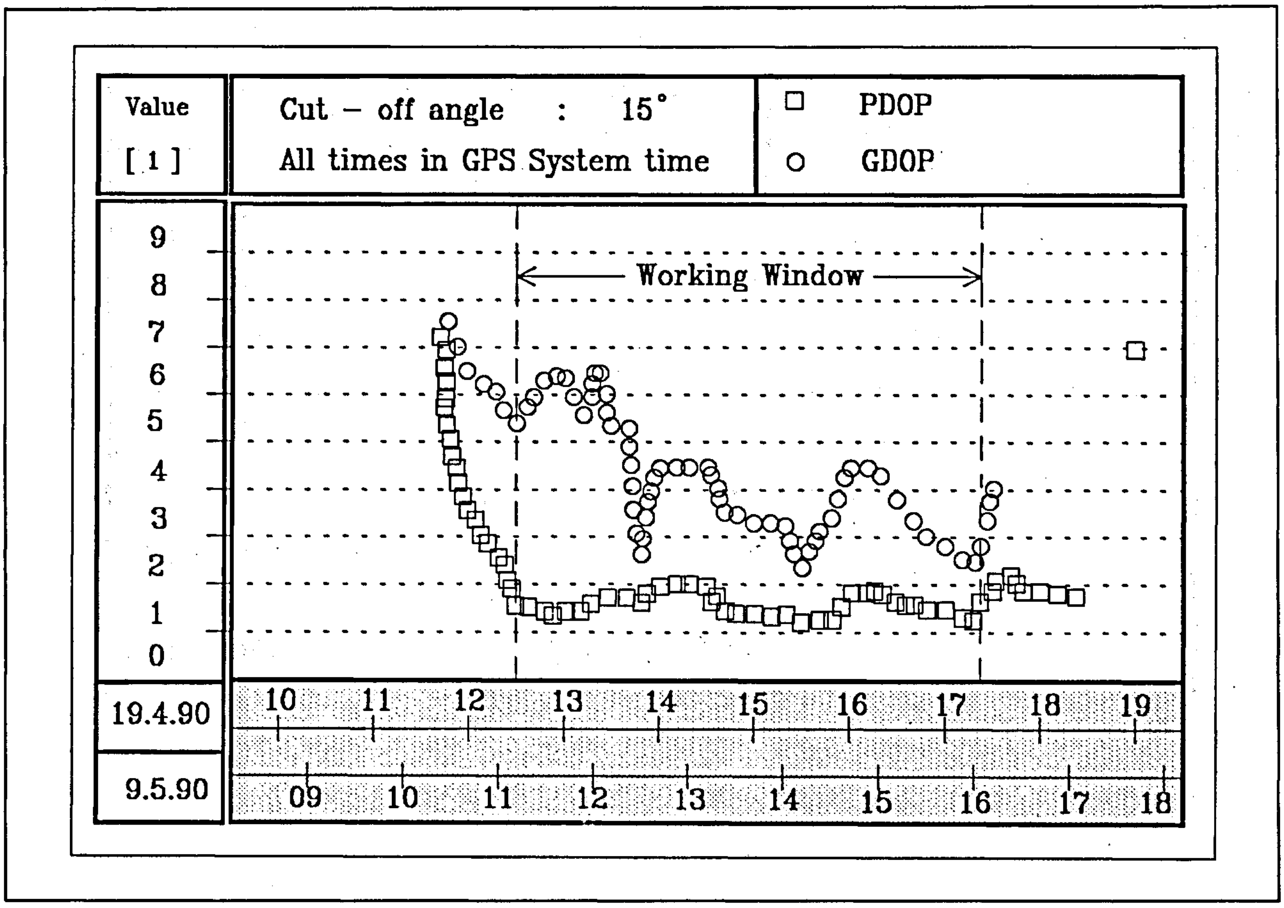


Figure 2.3.3 : GDOP and PDOP Values for the Constellation on the 19th of April 1990 and the 9th of May 1990

2.4 The Measuring Programs

2.4.1. The Measuring Program for the Heerbrugg Test Range

Following the basic test objective, namely to gather sufficient data to guarantee a successful ambiguity resolution, the decision was to take on each individual pillar five minutes worth of data. Due to the fact that WM102's and Trimble 4000 SLD Receivers were used L_1 and L_2 measurements could be taken. Although we knew that a single site occupation of about five minutes would provide sufficient data to resolve the ambiguities successfully, each site was occupied twice a day. The main reason was to demonstrate the capabilities of the "FARA" in a reoccupation scenario as outlined by various authors, e.g. by B. Remondi [REMONDI, 1990], by V. Ashkenazi [ASHKENAZI, 1989] or by G. Beutler et al [BEUTLER et al., 1989]. Therefore, the measuring program for each individual day of the Heerbrugg test included two measuring loops. One of the two receiver-antennas has been put up on pillar P00 before the first loop has been started. Then, each pillar was visited in sequence by the second receiver. On each individual pillar five minutes worth of data have been taken in a purely static mode. That means, no attempt was made to keep lock to the satellite signal while moving from one pillar to the next. Having finished the measurements on pillar P9 the receiver was taken back to pillar P0 where the second loop was started. Returning to the beginning for the second loop was required to equally space the second visits in time for all the pillars. The time between subsequent occupations of a particular site was in the order of about seventy to seventy five minutes. The decorrelation of participating satellite constellations should be almost complete after that time. The effect is almost the same if the satellites from the second occupation were observed and added to the satellite constellation of the first occupation. Obviously, the result is a much stronger geometry.

The very same measuring scenario has been repeated on each individual day for both WM102 and Trimble 4000 SLD receivers.

2.4.2 The Measuring Scenario for the "ad hoc" Testing

In order to be able to test the "FARA" on distances longer than "only" the three-kilometer-line as part of the Heerbrugg Test Range, four additional lines have been observed using WM102 receivers on the 28th of June 1990. The antenna separation distances were 5, 7, 10 and 12 kilometers. A ground truth for these lines is not available. Each line has been observed only once for approximately five to ten minutes.

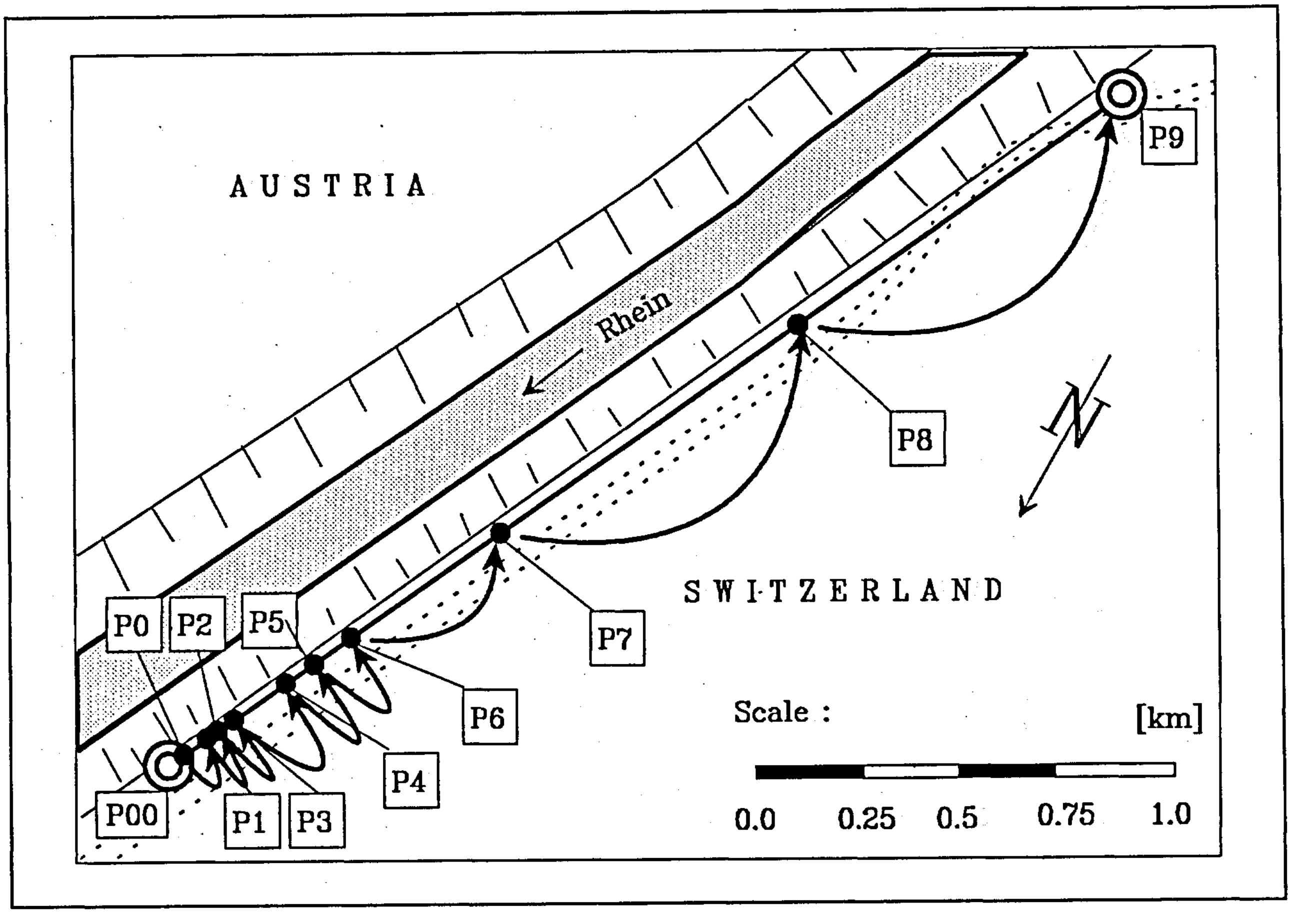


Figure 2.4.1 : Measuring Program for one Loop

2.5 Measurements with the WM102

Two WM102 receivers from Leica Heerbrugg Ltd., have been used on the Heerbrugg Test Range on the 19th and 20th of April. The WM102 is a dual band receiver which employes the P-code to reconstruct the phase of the L_2 carrier. Measurements have been recorded every 15 seconds. Each of the data points represents a compacted measurement over this 15 second time interval. The roving receiver was neither used in a static sense nor in a kinematic sense as far as its operation is concerned. The receiver was kept running all the time, but the antenna element was disconnected before moving to the next pillar and connected again as soon as the antenna was properly mounted on the next pillar. This procedure helped to save the time for initial satellite acquisition which

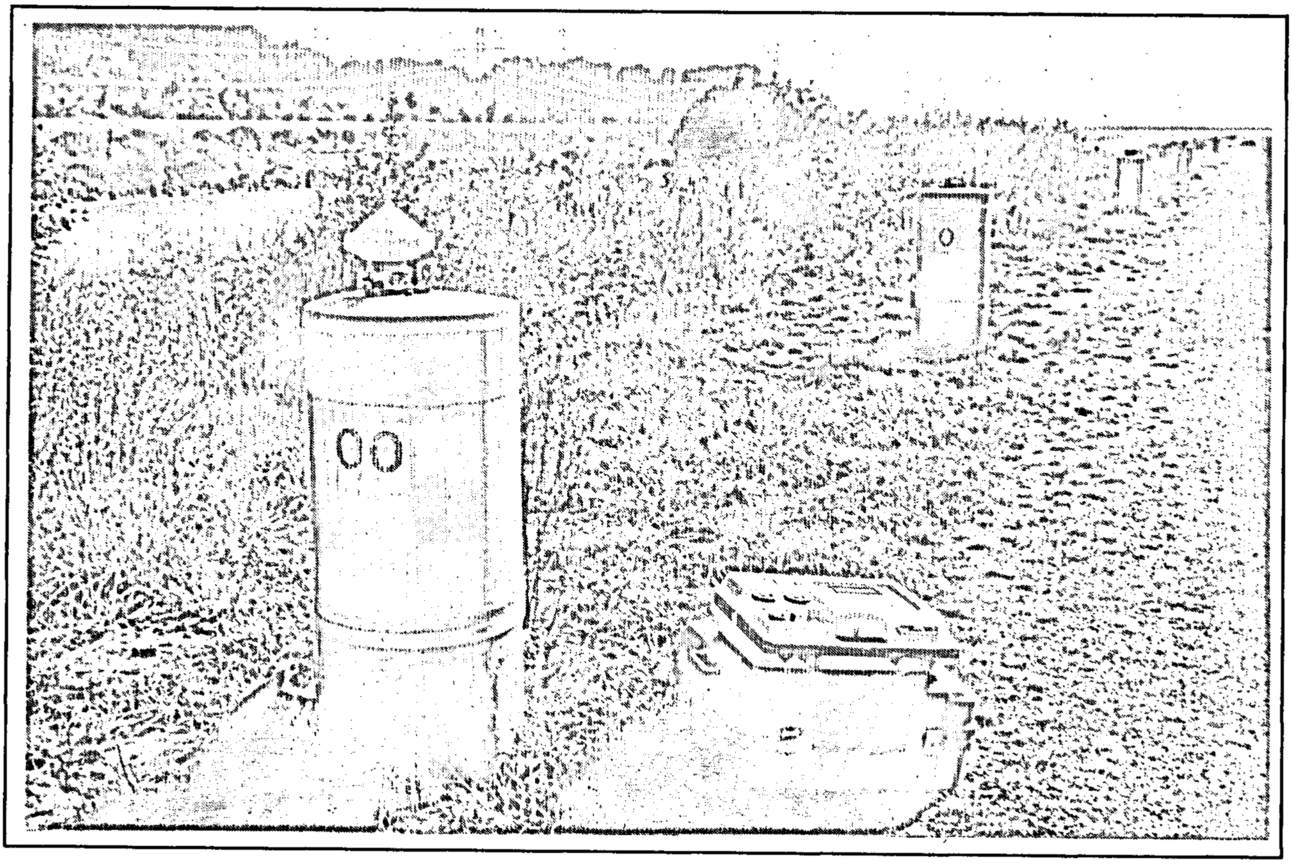


Figure 2.5.1 : The WM102 at Work on the Heerbrugg Test Range

would have been in the order of six to ten minutes at each individual site. Disconnecting and connecting the antenna enabled to track satellites after a few seconds time on the new site.

Table 2.5.2 gives a summary of the measurements which have been taken on the 19th of April with the WM102. The table shows the start time of each individual measuring block, the number of epochs (separated by 15 seconds), the total number of L_1 and L_2 measurements and the number of tracked satellites. All times are given in GPS system time. Due to the fact that there was a sixth satellite available in the second loop starting at 14:41, about 20 measurements more could be taken at each individual site in the second loop compared to the first loop. Measurements were transferred to a PC and recorded on files in the RINEX format using the POPS GPS Post Processing Software package [FREI, 1986]. The corresponding table for the 20th of April would not look too different from Table 2.5.2. Only the start times for individual site occupations are slightly different.

Table 2.5.2 : Measurements with the WM102 on the 19^{th} of April 1990

Date	Pillar	Start Time	# of Epochs	# of L_1 Observations	# of L_2 Observations	# of Satellites
[dd-mm-yy]		[hh:mm:ss]				
19-04-90	P00	12:53:45	350	-	-	-
19-04-90	P0	12:53:45	22	88	82	5
19-04-90	P1	13:00:15	25	96	89	5
19-04-90	P2	13:07:15	22	85	77	5
19-04-90	P3	13:14:15	26	96	88	5
19-04-90	P4	13:23:00	21	84	77	5
19-04-90	P5	13:30:45	23	90	80	5
19-04-90	P6	13:38:30	22	88	80	5
19-04-90	P7	13:46:15	21	85	71	6
19-04-90	P8	14:05:15	28	84	75	4
19-04-90	P9	14:14:30	24	96	88	5
19-04-90	P00	14:41:15	275	-	-	-
19-04-90	P0	14:41:15	22	88	80	5
19-04-90	P1	14:47:45	22	108	85	6
19-04-90	P2	14:54:00	22	102	92	6
19-04-90	P3	15:00:45	22	106	82	6
19-04-90	P4	15:07:30	21	93	54	6
19-04-90	P5	15:14:45	22	108	95	6
19-04-90	P6	15:21:30	21	102	90	6
19-04-90	P7	15:28:30	22	102	91	6
19-04-90	P8	15:35:45	22	84	73	5
19-04-90	P9	15:44:15	23	92	84	5

2.6 Measurements with the Trimble 4000 SLD

Two Trimble 4000 SLD receivers from the Swiss Federal Office of Topography, Wabern (L + T) were taken to the Heerbrugg Test Range on the 8^{th} and 9^{th} of May 1990. The Trimble 4000 SLD is a dual band receiver which tracks the L_2 signal with a squaring technique. The consequence is that instead of an ambiguity of an integer number of full cycles we have an integer number of half-cycles. Measurements have been taken every fifth second. The receiver moving from site to site was operated on the 8th of May in a strict static sense. It was initialized newly on each individual site. On the 9th of May the roving receiver was kept running all the time. However, in post-processing the data gathered on this day has been treated as if it would have been taken in a strict static sense.

Table 2.6.2 : Measurements with the Trimble 4000 SLD

Date	Pillar	Start Time	# of Epochs	# of Satellites
[dd-mm-yy]		[hh:mm:ss]		
9-5-90	P00	12:06:15	920	5
9-5-90	P0	12:06:15	59	5
9-5-90	P1	12:12:25	57	5
9-5-90	P2	12:19:30	56	5
9-5-90	P3	12:26:10	62	5
9-5-90	P4	12:33:50	51	5
9-5-90	P5	12:41:00	57	5
9-5-90	P6	12:48:00	67	5
9-5-90	P7	12:56:40	57	5
9-5-90	P8	13:07:05	56	5
9-5-90	P9	13:17:55	60	5
9-5-90	P00	13:39:05	950	5
9-5-90	P0	13:39:05	74	5
9-5-90	P1	13:46:15	59	5
9-5-90	P2	13:52:55	61	5
9-5-90	P3	14:00:05	63	5
9-5-90	P4	14:07:30	58	5
9-5-90	P5	14:14:10	59	5
9-5-90	P6	14:21:25	58	5
9-5-90	P7	14:29:15	58	5
9-5-90	P8	14:43:30	64	5
9-5-90	P9	14:53:20	59	5

Table 2.6.2 gives an overview of measurements taken with the Trimble 4000 SLD on the 9th of May. The corresponding table for the 8th of May, which is not presented here, looks similar in terms of number of satellites and number of observation epochs.

IV : 3. Processing Scenarios

Since the "FARA" can be employed for many different surveying scenarios e.g rapid static (one site occupation with a short observation period), or static with reoccupations (more than one site occupation for short observation periods), we wanted to test its performance at least with a few of these possible surveying scenarios.

All computations documented in the next section were performed using mostly identical models and parameters. Differences will be stated explicitly. The processing technicalities were :

(i) Observations :
- L_1 only and L_1 & L_2 phase measurements were processed (the L_2 phase measurements gathered with the Trimble 4000 SLD were acquired by a squaring technique, therefore half-cycle ambiguities had to be determined for this type of measurements).
- Code measurements were solely used to synchronize the receiver clocks to GPS time, but not to support the differential phase processing.
- The elevation cut-off angle was set to 0 degrees.

(ii) Stochastic Model :
- All correlations within a single baseline introduced by forming the double differenced observable were handled rigorously.

(iii) Stations :

- The coordinates for pillar P00, determined by a single point positioning using all available code measurements, were kept fixed for every line.

(iv) Satellite Orbits :
- The broadcast ephemerides were used to compute the satellite positions.

(v) Troposphere Model :
- The Saastamoinen model based on extrapolated meteo values given at sea level was used to correct for tropospheric refraction.

(vi) No ionosphere model was used.

IV : 4. Results Processing WM102 Data

4.1 Classical Ambiguity Resolution

Table 4.1.1 shows the results obtained by processing the full data sets (L_1 and L_2 data) from the 19^{th} of April using a classical ambiguity resolution strategy. The attempt to fix ambiguities failed for six out of the twenty baselines. Note that in the second loop starting at 14:41 the ambiguities of only two baselines could not be resolved, whereas the line P00 - P4, which failed, has significant less data than the neighboring lines. The second loop was observed with a much better satellite constellation (six satellites rather than 5 satellites for most of the points compared to loop 1). Classical ambiguity resolution strategies are based on the assumption that the real-valued ambiguities can be determined accurately (to better than 0.2 cycles) in the initial adjustment. The more measurements will be available the better the formal accuracies for the real-valued ambiguities will be. Therefore, classical ambiguity resolution techniques require an excellent satellite geometry for quite long measuring periods, and these techniques are not well suited for positioning taks with short site occupation times.

Table 4.1.1 : Classical Ambiguity Resolution with the WM102

Date	Time	Line	Resolved	# of L_1 Obs	# of L_2 Obs	# of Sats
[dd-mm-yy]	[hh:mm:ss]		[Yes/No]			
19-4-90	12:53:45	P00 - P0	Yes	88	82	5
19-4-90	13:00:15	P00 - P1	Yes	96	89	5
19-4-90	13:07:15	P00 - P2	No	85	77	5
19-4-90	13:14:15	P00 - P3	No	96	88	5
19-4-90	13:23:00	P00 - P4	Yes	84	77	5
19-4-90	13:30:45	P00 - P5	Yes	90	80	5
19-4-90	13:38:30	P00 - P6	Yes	88	80	5
19-4-90	13:46:15	P00 - P7	No	85	71	6
19-4-90	14:05:15	P00 - P8	No	84	75	4
19-4-90	14:14:30	P00 - P9	Yes	96	88	5
19-4-90	14:41:15	P00 - P0	Yes	88	80	5
19-4-90	14:47:45	P00 - P1	Yes	108	85	6
19-4-90	14:54:00	P00 - P2	Yes	102	92	6
19-4-90	15:00:45	P00 - P3	Yes	106	82	6
19-4-90	15:07:30	P00 - P4	No	93	54	6
19-4-90	15:14:45	P00 - P5	Yes	108	95	6
19-4-90	15:21:30	P00 - P6	Yes	102	90	6
19-4-90	15:28:30	P00 - P7	Yes	102	91	6
19-4-90	15:35:45	P00 - P8	No	84	73	5
19-4-90	15:44:15	P00 - P9	Yes	92	84	5

4.2 Processing the entire data sets using L_1 and L_2 data

The results of the first computations of "FARA 90" data using the "Fast Ambiguity Resolution Approach" is summarized in Table 4.2.1 . The term s_0 denotes the a posteriori standard deviation for a single-differenced phase measurement obtained by processing a particular line with fixed ambiguity values. The term T_S' represents the test statistic (see Part III for a detailed description), which is used to decide, whether the set of ambiguities belonging to the best a posteriori standard deviation of unit weight can be reliably used as the final solution. T_S' is simply the second best a posteriori variance of unit weight divided by the best a posteriori variance of unit weight. The larger T_S' the more significant is the separation of the two estimated a posteriori variances. Whenever the two a posteriori variances of unit weight must be considered identical from a statistical point of view, then the attempt to fix the ambiguities to integers has failed. Specifying a certain level of significance for such a compatibility test, a threshold for T_S' can be determined, which splits the full range of values for T_S' into a rejection range and an acceptance range. Whenever T_S' falls into the acceptance range it can be assumed on the selected level of significance, that the two a posteriori variances of unit weight are not compatible. That means, that the ambiguities belonging to the best a posteriori standard deviation can be taken as the correct solution. For the situations shown in Table 4.2.1 with a total of more than 150 observations and up to 13 unknown parameters for a particular line, the threshold is at a value of 1.4 ($\alpha = 5\%$). The values for T_S' labelled with a "*" have passed the statistical test and the corresponding ambiguities can therefore be taken as the final solution.

The column entitled "# of Amb. Sets" shows the number of integer ambiguity sets which finally had to be analyzed to obtain the corresponding a posteriori variances of unit weight. The terms ΔD and ΔH shown in the two last columns of Table 4.2.1 denote the difference between the estimated and the true slope distance resp. the difference between the true and the estimated height difference (see section IV : 2.2 for a detailed definition of ground truth). Let us now concentrate on the results shown in Table 4.2.1 . The standard deviations for single differenced phase measurements are in the order of 2 to 5 millimeters. For 9 of the 20 lines the "FARA" selected only one set of integer ambiguity parameters for the final computation run. That means, the correct solution has been found right away without providing any alternatives. The test statistics T_S' show the ratio between the best and the second best a posteriori variances of unit weight for the cases where alternative integer ambiguities had to be tested. The comparison with ground truth shown in the last two columns verifies what already can be read from the figures for T_S', namely, that all ambiguity parameters have been fixed to the correct integer values. The rms error (root-mean-square error) for a single slope distance is in the order of six millimeters, whereas the rms error for a height difference is in the order of 23 millimeters. The lines have been processed without applying corrections to reduce the impact of ionospheric refraction. This is nicely verified by the slope distances from pillar P00 to pillars P7, P8 and P9. It is known that not correcting for the ionospheric refraction introduces scale factors which are currently in the order of 3.5 to 4 ppm's for Heerbrugg for day-time observations. Applying these corrections would reduce the rms errors for a single slope distance to 3 millimeters. In order to verify these remarks the three longest lines have been re-processed using an ionospheric model derived by evaluating the differences of simultaneous L_1 and L_2 measurements of the P00 site (WILD et al., 1989). The corresponding results are shown in Table 4.2.2. Replacing the corresponding figures of Table 4.2.1 with the ones in Table 4.2.2 and performing a new calculation of rms errors yields three millimeters for a single slope distance and 19 millimeters for a single height difference. These results are excellent considering the short observation intervals.

The impressive strength of the determination of the correct sets of integer ambiguities leaves great expectations for a further reduction of occupation times from five minutes down to one-minute.

Table 4.2.1 : Processing the entire Data Sets using L_1 and L_2 Data

Date	Time	Line	s_0	$T_{s'}$	# of Amb. Sets	# L_1 Obs	# L_2 Obs	ΔD	ΔH
[dd-mm-yy]	[hh:mm:ss]	-	[mm]	-	-	-	-	[mm]	[mm]
19-4-90	12:53:45	P00 - P0	2.3	*	1	88	82	1.8	6
19-4-90	13:00:15	P00 - P1	3.4	72*	2	96	89	-4.2	-3
19-4-90	13:07:15	P00 - P2	2.9	210*	2	85	77	3.6	7
19-4-90	13:14:15	P00 - P3	4.0	128*	2	96	88	1.9	2
19-4-90	13:23:00	P00 - P4	2.6	*	1	84	77	0.1	10
19-4-90	13:30:45	P00 - P5	4.9	128*	2	90	80	3.6	-1
19-4-90	13:38:30	P00 - P6	2.5	*	1	88	80	0.0	3
19-4-90	13:46:15	P00 - P7	3.4	16*	4	85	71	-1.0	1
19-4-90	14:05:15	P00 - P8	3.1	529*	2	84	75	-2.4	-3
19-4-90	14:14:30	P00 - P9	2.9	*	1	96	88	-11.2	-22
19-4-90	14:41:15	P00 - P0	2.2	272*	2	88	80	4.3	0
19-4-90	14:47:45	P00 - P1	3.0	*	1	108	85	1.1	3
19-4-90	14:54:00	P00 - P2	2.5	225*	2	102	92	-0.3	-1
19-4-90	15:00:45	P00 - P3	2.4	*	1	106	82	-0.2	3
19-4-90	15:07:30	P00 - P4	3.0	49*	2	93	54	0.6	-55
19-4-90	15:14:45	P00 - P5	2.7	*	1	108	95	0.5	-1
19-4-90	15:21:30	P00 - P6	2.7	272*	2	102	90	-0.7	1
19-4-90	15:28:30	P00 - P7	3.0	*	1	102	91	-3.9	-14
19-4-90	15:35:45	P00 - P8	5.1	49	2	84	73	-14.5	-34
19-4-90	15:44:15	P00 - P9	4.4	*	1	92	84	-15.9	-74

Table 4.2.2 : Processing the entire Data Sets using L_1 and L_2 Data with a Ionosphere Model

Date	Time	Line	s_0	$T_{s'}$	# of Amb. Sets	# L_1 Obs	# L_2 Obs	ΔD	ΔH
[dd-mm-yy]	[hh:mm:ss]	-	[mm]	-	-	-	-	[mm]	[mm]
19-4-90	13:46:15	P00 - P7	3.4	16 *	4	85	71	3.9	10
19-4-90	14:05:15	P00 - P8	2.9	30 *	2	84	75	5.5	12
19-4-90	14:14:30	P00 - P9	2.9	*	1	96	88	-1.4	-2
19-4-90	15:28:30	P00 - P7	3.1	*	1	102	91	0.5	-10
19-4-90	15:35:45	P00 - P8	4.8	49 *	2	84	73	-6.7	-23
19-4-90	15:44:15	P00 - P9	3.6	*	1	92	84	-5.7	-58

4.3 Processing the entire data Setss using L_1 data only

The computations presented in section 4.2 have been repeated but this time using only L_1 measurements. The results shown in Table 4.3.1 differ considerably from those in Table 4.2.1. The ambiguities for 14 of the 20 lines could be resolved. The figures of T_S' were again used to decide whether the ambiguity fixing was done correctly or not. Taking a significance level of 95% and a degree of freedom of 80 the ambiguity resolution was considered to be successful if $T_S' > 1.46$. A closer inspection of Table 4.3.1 shows that nine lines have been successfully resolved in the second loop, whereas only five lines have been resolved in the first loop. Let us have a look at these results from a satellite geometry point of view. For this purpose, the actual measurement times for all the lines have been put into a modified version of the plot showing the Dilution of Precision (DOP) values (Figure 2.3.3). Figure 4.3.2 shows the successfully resolved lines with a black circle and the lines which could not be resolved with an open circle.

Looking at Figure 4.3.2 one could conclude that L_1 ambiguities can be resolved with approximately five minutes worth of data, as soon as the GDOP figure drops below five. Figure 4.3.2 demonstrates nicely, that the PDOP-values do not form an adequate means to predict the success of ambiguity resolution. However, it shows that without excellent satellite constellations (in minimum five satellites optimally distributed in space) ambiguity resolution with only five minutes L_1

observations is an illusion. There are methods to improve this situation. They will be covered in the reoccupation scenarios. From an accuracy point of view, there are no significant differences in using L_1 and L_2 measurements or L_1 measurements only provided the ambiguities could be resolved.

Table 4.3.1 : Processing the entire Data Sets using L_1 Data only

Date	Time	Line	s_0	$T_{s'}$	# of Amb. Sets	# L_1 Obs	# L_2 Obs	ΔD	ΔH
[dd-mm-yy]	[hh:mm:ss]	-	[mm]	-	-	-	-	[mm]	[mm]
19-4-90	12:53:45	P00 - P0	1.5	36*	9	88	-	-2.2	-2
19-4-90	13:00:15	P00 - P1	3.5	1.1	577	96	-	-	-
19-4-90	13:07:15	P00 - P2	3.6	1.0	901	85	-	-	-
19-4-90	13:14:15	P00 - P3	4.7	1.0	29	96	-	-	-
19-4-90	13:23:00	P00 - P4	1.9	14*	3	84	-	1.4	14
19-4-90	13:30:45	P00 - P5	6.5	1.2	85	90	-	-	-
19-4-90	13:38:30	P00 - P6	2.4	15*	4	88	-	-0.7	6
19-4-90	13:46:15	P00 - P7	3.2	2.1*	21	85	-	-2.5	-8
19-4-90	14:05:15	P00 - P8	1.7	1.2	5	84	-	-	-
19-4-90	14:14:30	P00 - P9	3.2	1.7*	25	96	-	-11.6	-17.9
20-4-90	14:41:15	P00 - P0	1.5	24*	7	88	-	3.3	-4
20-4-90	14:47:45	P00 - P1	2.8	6.1*	13	108	-	-0.3	-2
20-4-90	14:54:00	P00 - P2	2.2	8.9*	5	102	-	0.7	-1
20-4-90	15:00:45	P00 - P3	2.2	4.4*	3	106	-	-0.7	3
20-4-90	15:07:30	P00 - P4	2.8	5.7*	19	93	-	-0.9	-4
20-4-90	15:14:45	P00 - P5	2.5	12*	5	108	-	-1.0	-5
20-4-90	15:21:30	P00 - P6	2.2	19*	5	102	-	-3.2	-3
20-4-90	15:28:30	P00 - P7	2.4	141*	3	102	-	-6.5	-15
20-4-90	15:35:45	P00 - P8	5.5	1.4	23	84	-	-	-
20-4-90	15:44:15	P00 - P9	3.2	10*	3	92	-	-13.8	-69

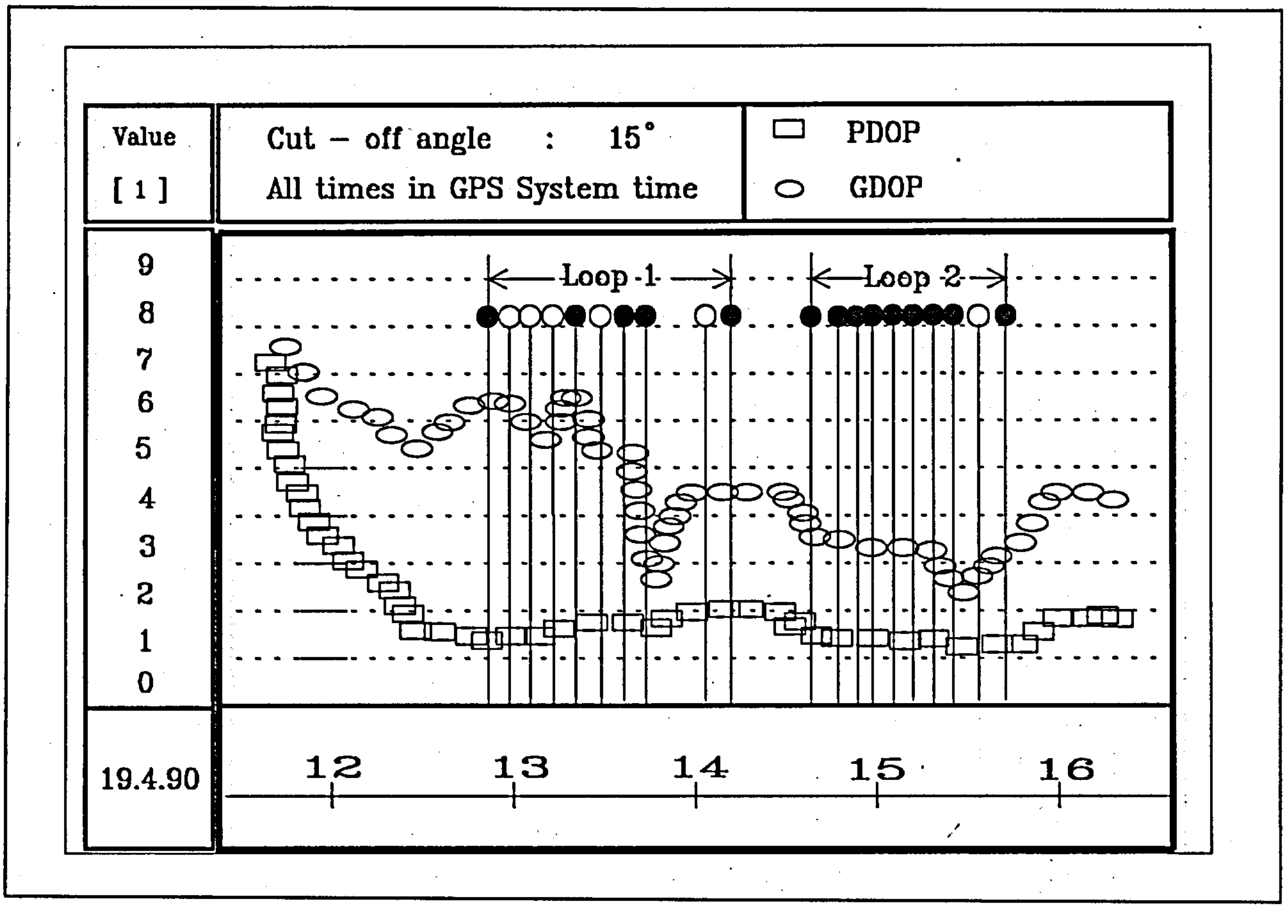


Figure 4.3.2 : *Satellite Geometry and Ambiguity Resolution*

4.4 Processing one minute worth of L_1 and L_2 data

Table 4.4.1 presents the results when processing one minute worth of L_1 and L_2 data. For each individual line there are 30 to 50 measurements available gathered over a one minute period (5 observation epochs only). The resulting a posteriori standard deviations for single-differenced phase measurements are within 2 to 5 millimeters. The figures for the test statistics T_S' are not smaller than 2.3 (line P00 - P4 in loop 2). The acceptance threshold is at 2.1 ($\alpha = 5\%$, $f = 20$). Therefore, all the resolved ambiguities can be accepted. A closer inspection of the comparison with ground truth verifies, that the ambiguity resolution is indeed correct. Taking into account the 4 ppm scale factor due to ionosphere, the computation of the rms error for a single slope distance yields 2.8 millimeters. The rms error of a single height difference is in the order of 18 millimeters (including corrections for ionosphere). Average processing time for the ambiguity search using an MS-DOS PC with a 80386/16 MHz processor was about 2 seconds. In comparison of this processing scenario with the L_1 only scenario in the previous section it is interesting to note that using L_1 and L_2 data there are no difficulties to resolve ambiguities at all. This demonstrates again the main difference between L_1 only and L_1 and L_2 based position determinations. Using L_1 data only, sufficient information has to be provided through the satellite geometry itself; whereas using L_1 and L_2 data the information is provided by the geometrical relationship between simultaneous L_1 and L_2 measurements.

Table 4.4.1 : Processing one Minute worth of L_1 and L_2 Data

Date	Time	Line	s_0	T_s'	# of Amb. Sets	# L_1 Obs	# L_2 Obs	ΔD	ΔH
[dd-mm-yy]	[hh:mm:ss]	-	[mm]	-	-	-	-	[mm]	[mm]
19-4-90	12:53:45	P00 - P0	2.1	98.0*	3	20	20	0.4	6
19-4-90	13:00:15	P00 - P1	2.3	998*	3	20	20	-3.0	-3
19-4-90	13:07:15	P00 - P2	2.1	73*	3	20	20	3.9	10
19-4-90	13:14:15	P00 - P3	2.1	576*	2	19	18	2.8	2
19-4-90	13:23:00	P00 - P4	2.2	942*	2	20	20	0.9	9
19-4-90	13:30:45	P00 - P5	1.9	331*	2	20	20	2.7	0
19-4-90	13:38:30	P00 - P6	2.4	243*	2	20	20	0.2	5
19-4-90	13:46:15	P00 - P7	1.6	*	1	20	19	-1.9	-2
19-4-90	14:05:15	P00 - P8	3.0	3.2*	5	15	15	-3.1	0
19-4-90	14:14:30	P00 - P9	1.9	841*	2	20	20	-14.1	-24
20-4-90	14:41:15	P00 - P0	1.8	416*	2	20	19	4.4	-1
20-4-90	14:47:45	P00 - P1	2.8	225*	2	25	25	0.2	4
20-4-90	14:54:00	P00 - P2	1.7	201*	2	25	25	-2.0	-1
20-4-90	15:00:45	P00 - P3	1.9	529*	2	25	23	-1.9	3
20-4-90	15:07:30	P00 - P4	3.1	2.3*	9	20	15	-4.3	-10
20-4-90	15:14:45	P00 - P5	2.3	198*	2	25	22	2.8	-3
20-4-90	15:21:30	P00 - P6	2.6	198*	2	25	23	-0.2	-1
20-4-90	15:28:30	P00 - P7	2.4	625*	2	25	25	-3.9	-1
20-4-90	15:35:45	P00 - P8	3.9	100*	2	20	20	-13.1	-30
20-4-90	15:44:15	P00 - P9	4.3	94*	2	20	20	-15.0	-75

4.5 Reoccupation with one minute worth of L_1 and L_2 data

Table 4.5.1 shows the results obtained by processing four selected lines from the 19th of April in a reoccupation scenario. One minute worth of L_1 and L_2 data for the two occupations of a specific site have been combined in one computation run. All ambiguities have been resolved successfully. The acceptance threshold for $T_{S'}$ is 1.6 (α = 5%, f = 65). The time required to perform the ambiguity search was not longer than two seconds using a 80386/20MHz PC. The test statistic T_S' indicates how strong the determination of position is in applying this reoccupation technique. The comparison to ground truth has not changed compared to the previously presented processing scenarios. This is not surprising, because as soon as ambiguities can be resolved to integers the accuracies have to be comparable on the centimeter level considering the almost identical satellite geometry. For these reoccupation scenarios it is expected, that the a posteriori standard deviations will be larger than for the single occupation scenarios. Returning to a site which was visited once half an hour ago the atmospheric conditions at this site might have changed. This must result in an increased a posteriori standard deviation of unit weight for a combined computation of these two occupations.

Table 4.5.1 : Reoccupation with one Minute worth of L1 and L2 Data

Date	Loop	Line	s_0	$T_{s'}$	# of Amb. Sets	# L_1 Obs	# L_2 Obs	ΔD	ΔH
[dd-mm-yy]	-	-	[mm]	-	-	-	-	[mm]	[mm]
19-4-90	1&2	P00 - P0	2.2	457*	2	40	39	1.5	5
19-4-90	1&2	P00 - P3	2.3	625*	2	44	41	-0.5	1
19-4-90	1&2	P00 - P7	2.3	296*	2	45	44	-2.9	-14
19-4-90	1&2	P00 - P9	7.0	48*	2	40	40	-14.0	-39

4.6 Reoccupation with the entire data sets using L_1 data only

After processing the entire data sets for a single occupation using L_1 data only (see section 4.3) we processed this data again, but this time imitating a reoccupation scenario. The data gathered in loop 1 and loop 2 have been combined in one computation run. The results are shown in Table 4.6.1. The acceptance threshold for $T_{S'}$ is 1.3 (α = 5%, f = 180). A screening through the figures for T_S' shows, that there were no difficulties to resolve the ambiguities for the four arbitrarily selected lines. Therefore, a further reduction of the occupation time seemed to be feasible. The corresponding results are shown in the next section. Note, that the ambiguity search came up with at most three different sets of integer ambiguities which had to be analyzed in a further computation step in terms of resulting a posteriori variances of unit weight.

Table 4.6.1 : *Reoccupation with the entire Data Sets using L_1 Data only*

Date	Loop	Line	s_0	$T_{S'}$	# of Amb. Sets	# L_1 Obs	# L_2 Obs	ΔD	ΔH
[dd-mm-yy]	-	-	[mm]	-	-	-	-	[mm]	[mm]
19-4-90	1&2	P00 - P0	1.8	*	1	176	-	-0.1	1
19-4-90	1&2	P00 - P3	3.6	117*	2	202	-	-0.5	3
19-4-90	1&2	P00 - P7	3.4	*	1	187	-	-4.5	-9
19-4-90	1&2	P00 - P9	5.9	32*	3	188	-	-12.1	-33

4.7 Reoccupation with one minute worth of L_1 data

One minute worth of L_1 data for four selected lines were combined in one computation run. Table 4.7.1 shows the results obtained. First of all, the ambiguities could be resolved for all the lines on a very high level of significance. The threshold for T_S' is at a value of 2.1 (α = 5 %, f = 20). The interesting point is, that up to 22,000 different integer ambiguity sets had to be evaluated in terms of resulting a posteriori standard deviations. It is easy to understand that the reduction in occupation time had to be balanced by a longer computation time. To evaluate the lines P00 - P9 and P00 - P7 a computation time of three seconds was required, for line P00 - P0 44 seconds and for line P00 - P3 almost five minutes using a 80386/16MHz PC. The trade-off between computation time and occupation time is undoubtedly a matter of considerations regarding commercial aspects. However, there is no doubt, that ambiguities can be resolved using twice one minute worth of L_1 measurements provided the two occupations of the identical sites are well separated in time. The results above have been computed using a first implementation of the "FARA". It has been shown in the meantime, that the search strategy can be improved considerably, so that the actual computation time needed is almost independent of the number of alternative ambiguity combinations to be evaluated.

Table 4.7.1 : Reoccupation with one Minute wort of L_1 Data

Date	Loop	Line	s_0	$T_{s'}$	# of Amb. Sets	# L_1 Obs	# L_2 Obs	ΔD	ΔH
[dd-mm-yy]	-	-	[mm]	-	-	-	-	[mm]	[mm]
19-4-90	1&2	P00 - P0	1.7	49*	3399	40	-	-0.3	2
19-4-90	1&2	P00 - P3	2.0	36*	22894	44	-	-0.9	3
19-4-90	1&2	P00 - P7	2.2	54*	241	45	-	-4.4	-12
19-4-90	1&2	P00 - P9	5.9	5.3*	18103	40	-	-12.1	-36

4.8 Looking for the limits of the "FARA"

4.8.1 Limits in terms of satellite geometry

The line P00 - P7 has been observed on the 19th of April 1990 under a six satellite constellation. In order to be able to study the impact of satellite geometry all possible satellite combinations with at least four satellites for this particular line were evaluated in terms of the resulting GDOP and PDOP values (see Table 4.8.1). Out of these twelve different combinations the scenarios with the numbers 1, 5, 6, 11 and 12 have been actually analyzed using site occupation times of 60, 30 and 15 seconds. The results are presented in Table 4.8.2 .

Table 4.8.1 : GDOP values for the different satellite combinations for line P00 - P7

Scenario No.	# of Sats	GDOP	PDOP	Satellites
1	4	8.0	1.7	9 12 18 19
2	4	5.4	1.9	3 12 18 19
3	4	4.9	1.8	3 9 12 19
4	4	4.2	2.1	3 9 18 19
5	4	3.2	1.8	3 9 12 18
6	5	5.3	4.8	9 12 13 18 19
7	5	5.2	1.6	3 12 13 18 19
8	5	4.5	1.7	3 9 12 13 19
9	5	2.9	1.7	3 9 12 18 19
10	5	2.9	1.6	3 9 13 18 19
11	5	2.9	1.6	3 9 12 13 18
12	6	2.7	1.5	3 9 12 13 18 19

Browsing through Table 4.8.2 shows that the ambiguities could not be resolved for all the satellite combinations successfully. The scenarios with only four satellites do not provide necessarily sufficient information to resolve the initial phase ambiguities to integers. The values for the test statistics T_S, labelled with a "*" passed the test. The values in brackets are the actual test thresholds. Definitely not enough information is provided, if the site occupation time is reduced to 30 seconds and less using four satellite constellations. However, it is interesting to note that the constellations with five and more satellites provide in all cases sufficient data to perform a reliable resolution to integers using only 15 seconds worth of L_1 and L_2 data (only 2 epochs). This can be seen as an additional indication, that ambiguities can be resolved almost instantaneously provided a satellite constellation with a sufficient quality is available. A few theoretical developments in this respect will be presented in Part V. A comparison of scenarios 6 and 11 in terms of the differences of the results compared to ground truth confirms the rule that the poorer the GDOP and PDOP values the less accurate the position estimates will be. The ability to resolve the ambiguities seems not to be affected by this slightly worse satellite geometry.

Table 4.8.2 : Limits in terms of satellite geometry. Line P00 - P7 was analyzed under different satellite constellations. The computations were carried out with 15, 30 and 60 seconds worth of L_1 and L_2 measurements.

Number	Observ. Time	Scenario No.	s_0	$T_{s'}$	# of Amb. Sets	# L_1 Obs	# L_2 Obs	ΔD	ΔH
-	[s]	-	[mm]	-	-	-	-	[mm]	[mm]
1	60	1	1.9	4.5* (2.1)	5	15	15	-25.2	-19.3
2	30	1	1.1	4.6* (3.2)	4	9	9	-32.4	-17.5
3	15	1	1.1	2.2 (9.3)	9	6	6	-29.4	-16.1
4	60	5	2.2	1.4 (2.1)	5	15	15	-4.1	-15.3
5	30	5	1.3	1.7 (3.2)	7	9	9	1987	370
6	15	5	1.4	1.1 (9.3)	6	6	6	1987	370
7	60	6	2.3	282* (1.8)	2	20	20	-13.2	-19.7
8	30	6	2.0	1339* (2.5)	3	12	12	-15.2	-18.1
9	15	6	1.7	772* (5.0)	3	8	8	-15.5	-16.6
10	60	11	2.3	5.8* (1.8)	5	20	20	-3.8	-15.9
11	30	11	1.9	7.6* (2.5)	6	12	12	-3.8	-13.5
12	15	11	1.5	1276* (5.0)	5	8	8	-3.8	-11.7
13	60	12	2.4	665* (1.7)	2	25	25	-4.0	-17.6
14	30	12	2.3	371* (2.3)	3	15	15	-4.1	-15.6
15	15	12	2.1	1030* (3.8)	7	10	10	-3.1	-13.8

4.8.2 Limits in terms of antenna separation distance

Following the basic test objective (as outlined in section 3.8.2) four lines were analyzed using 60 seconds worth of L1 and L2 data. Because no ground truth is available all the available measurements (almost 10 minutes worth of data) were processed to give the reference solution which can be used to test whether the ambiguity resolution with 60 seconds worth of data was successful or not. Table 4.8.3 shows the results of these computations.

In contrary to the analysis of the data stemming from the "FARA 90" campaign the measurements for these four lines were corrected for refraction effects caused by the ionosphere. All the measurements of all four lines were used to derive a model for the ionosphere. This was performed using a built-in feature of the Bernese Second Generation GPS Software Package Version 3.2 (WILD et al., 1989). Selected computations were repeated without applying any ionospheric corrections. These results are summarized in Table 4.8.4 .

Table 4.8.3 : *Limits in terms of antenna separation distance (corrections applied for ionospheric refraction)*

Line	Observ. Time	# of Sats	s_0	$T_{s'}$	# of Amb. Sets	# L_1 Obs	# L_2 Obs	Slope Distance
-	[s]	-	[mm]	-	-	-	-	[m]
1	all	6	6.3	43*	2	262	262	5555.466
1	60	6	5.4	96*	2	24	24	5555.468
2	all	5	3.3	*	1	85	85	7653.745
2	60	5	4.0	26*	2	15	15	7653.737
3	all	4	3.6	228*	2	105	105	10091.439
3	60	4	2.0	580*	2	13	13	10091.448
4	all	5	6.3	81*	2	106	106	12236.922
4	60	5	3.8	14*	2	19	19	12236.921
4	30	5	3.5	100*	4	12	12	12236.918

Seemingly, all the ambiguities for all the lines could be resolved successfully. Even on the ten kilometer line with a four satellite constellation ambiguities were resolved with one minute worth of data. For the fourth line an attempt was made to further reduce the site occupation time. The result was that even 30 seconds of data were sufficient to resolve the ambiguities. It is remarkable how little the results using 60 seconds of data differ from the results enabling all available data. The maximum difference is in the order of 1 centimeter. The test statistics $T_{s'}$ never even reach the neighborhoods of the critical value of $T_{s'}$ which is around 2.6 (a = 5%, f = 13). The search procedure accepted at most four alternative integer ambiguity combinations for a further evaluation in terms of resulting a posteriori variance of unit weight. This fact reflects again the strong geometry.

Table 4.8.4 represent actually the same data sets as in Table 4.8.3 with the only difference, that no corrections for ionospheric refraction were applied. As the results demonstrate the "FARA" worked successfully. The comparison of the slope distances for these two different computation models shows that the distances without corrections for the effects of ionosphere are shorter than the ones with corrections applied. The scale factor varies quit considerably for the four lines from 1 ppm to 3.2 ppm, which is within the expectations. However, these results indicate, that it will be possible to resolve ambiguities even for baselines in the order of 10 to 20 kilometers without special efforts to reduce systematic disturbances caused by either ionosphere or troposphere.

Table 4.8.4 : Limits in terms of antenna separation distance (no corrections applied for ionospheric refraction)

Line	Observ. Time	# of Sats	s_0	T_s'	# of Amb. Sets	# L_1 Obs	# L_2 Obs	Slope Distance
-	[s]	-	[mm]	-	-	-	-	[m]
1	all	6	6.7	39*	2	262	262	5555.448
2	all	5	3.0	*	1	85	85	7653.736
3	all	4	4.8	136*	2	105	105	10091.428
4	all	5	5.3	110*	2	106	106	12236.912

4.8.3 Limits in terms of site occupation time

Again the line P00 - P7 was analyzed, this time with the emphasis on the amount of data necessary to resolve the initial phase ambiguities to integers. For this purpose L_1 and L_2 data as well as L_1 data only were processed. The results are shown in Table 4.8.5 .

Table 4.8.5 : Limits in terms of site occupation time

Line	Observ. Time	# of Sats	s_0	$T_{s'}$	# of Amb. Sets	# L_1 Obs	# L_2 Obs	ΔD	ΔH
-	[s]	-	[mm]	-	-	-	-	[mm]	[mm]
P00 - P7	60	6	2.4	665*	2	25	25	-4.0	-17.6
P00 - P7	45	6	2.4	636*	3	20	20	-4.1	-17.0
P00 - P7	30	6	2.3	371*	3	15	15	-4.1	-15.6
P00 - P7	15	6	2.1	1035*	7	10	10	-3.1	-13.8
P00 - P7	240	6	2.4	2.9*	10	73	-	-6.7	-14.9
P00 - P7	180	6	2.4	2.6*	27	61	-	-6.0	-14.6
P00 - P7	120	6	2.4	1.2	91	45	-	-4.7	-14.7
P00 - P7	60	6	2.2	1.0	3474	25	-	-5.8	-15.6
P00 - P7	30	6	2.1	1.0	80594	15	-	1128	900

Using L_1 and L_2 data the ambiguities could be resolved using only 15 seconds worth of data (that corresponds to two epochs), whereas three minutes worth of L_1 data are required to resolve the corresponding ambiguities successfully. The test statistic $T_{s'}$ is used to decide whether the set of integer ambiguities yielding the smallest a posteriori variance of unit weight can be accepted as a unique solution. For the case where three minutes worth of L_1 data were processed the critical value for $T_{s'}$ is around 1.6 ($\alpha = 5\%$, $f = 53$) and around 1.75 ($\alpha = 5\%$, $f = 37$) for two minutes worth of data. Therefore two minutes worth of L1 data are <u>not</u> sufficient to reliably resolve the ambiguities to integers according to the applied decision criteria. It is again worthwhile, that, provided the ambiguities can be resolved, there are no significant differences in the quality of the results between L1 only and L1 and L2 computations.

The results above are provoking again two essential questions, namely :

(i) Is it possible to predict by means of an a priori analysis whether the ambiguity resolution for a certain satellite constellation will be successful or not ?
and

(ii) Is it possible to resolve ambiguities almost instantaneously ?

These two questions will be dealt with later on. However, the bottom line of this particular test is, that ambiguities can be resolved with less than one minute worth of L_1 and L_2 data provided a reasonable satellite constellation is available. L_1 data do not provide sufficient information to resolve ambiguities with less than three to five minutes worth of data. It is well understood, that these statements are only valid for the satellite constellations and the environmental conditions observed for this particular test and should therefore be carried over cautiously to predict the behavior of other tests.

IV : 5. Results Processing Trimble 4000 SLD Data

Regarding the presentation of results obtained by analyzing Trimble 4000 SLD data, only selected processing scenarios will be included in this document. In general, there are no differences comparing Trimble 4000 SLD and WM102 results as far as the resolution of initial phase ambiguities is concerned. Emphasis is put on those scenarios representing the key scenarios for the "FARA" and its applications for GPS positioning techniques. It is in no way the intention of these tests to compare the two receiver families in terms of performance. The main objective to use two different receiver types was the fact that one of them acquires the second frequency with a squaring technique (Trimble 4000 SLD) whereas the other type of receiver (WM102) uses the P-Code to reconstruct the carrier.

The following results are included in this report regarding the data analysis of Trimble 4000 SLD data :

(i) Processing the entire data sets using L_1 data only,

(ii) Processing one minute worth of L_1 and L_2 data and

(iii) Processing one minute worth of L_1 data in a reoccupation scenario

5.1 Processing the entire data sets using L_1 data only

The lines P00 - P0, P00 - P3, P00 - P7 and P00 - P9 were analyzed with the entire data sets observed on the 9th of May 1990 for both measuring loops. The results are shown in Table 5.1.1 .

Table 5.1.1 : Processing the entire Data Sets using L_1 Data only

Date	Time	Line	s_0	T_s'	# of Amb. Sets	# L_1 Obs	# L_2 Obs	ΔD	ΔH
[dd-mm-yy]	[hh:mm:ss]	-	[mm]	-	-	-	-	[mm]	[mm]
9-5-90	12:07:00	P00 - P0	1.5	146*	2	236	-	-1.2	-2
9-5-90	12:28:00	P00 - P3	1.7	*	1	248	-	-2.9	-7
9-5-90	12:58:00	P00 - P7	1.6	14*	2	228	-	-2.4	17
9-5-90	13:20:00	P00 - P9	2.5	20*	3	240	-	-19.3	10
9-5-90	13:40:00	P00 - P0	1.1	*	1	291	-	4.2	-4
9-5-90	14:02:00	P00 - P3	1.6	*	1	252	-	14.7	-28
9-5-90	14:31:00	P00 - P7	1.8	18*	4	231	-	3.3	26
9-5-90	14:55:00	P00 - P9	1.9	1.2	9	236	-	-	-

The ambiguities for all processed lines could be resolved apart from line P00 - P9 in the second loop. Considering the satellite geometry for this particular line (GDOP > 4) it is not surprising , that line P00 - P9 could not be resolved. The same behavior was also noticed analyzing WM102 data. The a posteriori standard deviations of unit weight are all in the order of about two millimeters. The comparison to ground truth yields similar differences as obtained with the WM102.

5.2 Processing one minute worth of L_1 and L_2 data

All the measurements taken on the 9th of May have been processed in a rapid static sense using one minute worth of L1 and L2 data. No results could be obtained in processing the line P00 - P4 in the first loop. The reasons for this situation are still being investigated. Table 5.2.1 shows the results obtained.

Table 5.2.1 : Processing one Minute worth of L_1 and L_2 Data

Date	Time	Line	s_0	$T_{s'}$	# of Amb. Sets	# L_1 Obs	# L_2 Obs	ΔD	ΔH
[dd-mm-yy]	[hh:mm:ss]	-	[mm]	-	-	-	-	[mm]	[mm]
9-5-90	12:07:00	P00 - P0	1.7	49*	10	52	41	0.1	0
9-5-90	12:13:00	P00 - P1	2.1	36*	4	52	52	-3.2	0
9-5-90	12:21:00	P00 - P2	1.7	121*	3	52	52	-2.6	0
9-5-90	12:28:00	P00 - P3	1.7	145*	5	52	52	-3.0	-8
9-5-90	12:35:00	P00 - P4	-	-	-	-	-	-	-
9-5-90	12:42:00	P00 - P5	2.7	216*	3	52	52	-5.5	16
9-5-90	12:50:00	P00 - P6	2.7	43*	2	52	52	-8.8	19
9-5-90	12:58:00	P00 - P7	2.0	12*	5	52	30	1.6	11
9-5-90	13:09:00	P00 - P8	2.6	65*	3	52	52	-0.1	10
9-5-90	13:20:00	P00 - P9	2.8	22*	12	52	52	-12.9	8
9-5-90	13:40:00	P00 - P0	1.5	196*	3	52	46	1.8	6
9-5-90	13:48:00	P00 - P1	1.4	441*	2	52	50	2.7	-2
9-5-90	13:54:00	P00 - P2	1.3	56*	2	52	52	8.5	-10
9-5-90	14:02:00	P00 - P3	1.4	30*	2	52	52	19.9	-47
9-5-90	14:09:00	P00 - P4	1.8	18*	4	52	52	-0.5	28
9-5-90	14:16:00	P00 - P5	1.5	61*	4	52	52	-7.8	59
9-5-90	14:23:00	P00 - P6	1.3	130*	4	52	52	2.2	18
9-5-90	14:31:00	P00 - P7	1.6	79*	4	52	52	4.5	14
9-5-90	14:45:00	P00 - P8	1.3	41*	5	52	52	-5.7	31
9-5-90	14:55:00	P00 - P9	2.1	3.6*	9	52	52	-10.6	26

The ambiguities for all the processed lines could be reliably resolved. The standard deviations of unit weight are all below three millimeters and the values for the test statistic $T_{s'}$ is always bigger than 3.6. The critical value for $T_{s'}$ is at about 1.4 ($\alpha = 5\%$, $f = 100$). The rms error for a single slope distance is in the order of 7.3 millimeters. The corresponding rms error for the height differences is in the order of 23 millimeters.

5.3 Reoccupation with one minute worth of L_1 data

The last computation scenario with Trimble data which will be covered in this report is a reoccupation scenario using two blocks of one minute worth of L1 data separated in time by more than 90 minutes.

Table 5.3.1 : Reoccupation with one Minute wort of L_1 Data

Date	Loop	Line	s_0	$T_{s'}$	# of Amb. Sets	# L_1 Obs	# L_2 Obs	ΔD	ΔH
[dd-mm-yy]	-	-	[mm]	-	-	-	-	[mm]	[mm]
9-5-90	1&2	P00 - P0	1.2	96*	1887	104	-	0.0	0
9-5-90	1&2	P00 - P3	2.2	23*	651	104		-3.8	0
9-5-90	1&2	P00 - P7	1.9	35*	13697	104		0.1	14
9-5-90	1&2	P00 - P9	3.7	6.2*	11920	104		-14.8	12

The ambiguity resolution was successful too in this processing scenario. Note that a huge number of integer ambiguity sets were accepted in the search and as a consequence all these accepted combinations had to be analyzed in terms of resulting a posteriori variance of unit weight. This points again to the fact that scenarios with only L1 data are much weaker in terms of the search performance compared to the scenarios with L_1 and L_2 data. The reason is that the confidence region (n-dimensional hyperellipsoid) is much larger in the L_1 only case. The geometrical strength of simultaneous L_1 and L_2 measurements is missing.

IV : 6. Summary and Conclusions

Processing the "FARA" 90 campaign and the "ad hoc" test data has shown, that :

(i) Ambiguities for baselines of up to 13 kilometers can be resolved reliably with one minute worth of L_1 and L_2 data.

(ii) Computations for selected baselines with less than one minute worth of L_1 and L_2 data have indicated that an almost instantaneous ambiguity resolution will be feasible even with five-to six-satellite-constellations.

(iii) The applied test statistic T_s, never failed to give the right answer whether or not the selected ambiguities were correct.

(iv) The required computation times to perform the ambiguity search is reasonable (0 to 3 seconds on a 16 MHz DOS - computer with one minute worth of L_1 and L_2 data). Further improvements are possible.

(v) The "FARA" does not only work for rapid static positioning approaches but also for reoccupation scenarios.

(vi) There are no significant differences between a squaring-type receiver and a P - Code receiver as far as the performance of ambiguity resolution is concerned.

(vii) The rms errors for a single slope distance is in the order of 3 to 7 millimeters.

(viii) The rms error for a single height difference is in the order of 15 to 23 millimeters.

(ix) The results of processing L_1 data in rapid static scenarios seem to indicate, that satellite constellations with a GDOP < 5 or at least five satellites provide sufficient information to resolve ambiguities with three to five minutes worth of data.

(x) The reoccupation scenarios allow to resolve ambiguities with only one minute worth of L_1 data. Even four- satellite-constellations seem to provide sufficient information to resolve the ambiguities successfully with two very short site occupations.

(xi) Baselines up to 15 kilometers can still be handled by the "FARA". Sophisticated approaches to correct for ionospheric refraction are not required.

(xii) An accurate and reliable strategy to predict the site occupation time observing a particular satellite constellation is mandatory if an exploitation of the "FARA" for high performance surveying is considered. Such a strategy has clearly to be available on site.

(xiii) There are no significant differences in accuracy between a five minute occupation time and a one minute occupation time. In addition, if ambiguities can be resolved and the effects of ionospheric refraction (scale factor) can be corrected, there are no significant differences between the results of L_1 only and L_1 & L_2 data sets. The main effect in handling the L_2 measurements correctly results in a much faster ambiguity resolution than with L_1 measurements only, whereas the improvement in accuracy is marginal.

In summary, the "FARA" fulfills the initial specifications and opens up a variety of applications for high performance high accuracy surveys with GPS. The utilization of the "FARA" for rapid static positioning with the ability to resolve initial phase ambiguities with only one minute worth of L_1 and L_2 data represents undoubtedly a very potent positioning technique for detail surveys. Considering the advantages of such a surveying technique, namely the operational flexibility, the short site occupation time and the accuracy in position on the sub-centimeter level, leads to the conclusion that this technique will become a major competitor to classical equipment. A further improvement of the available satellite constellations leaves great expectations in view of an almost instantaneous ambiguity resolution capability. Initial investigations have shown, that there are no reasons why an ambiguity resolution with just one or two epochs (only a few seconds apart) should not be possible. It is hard to imagine a single surveying application which will not be affected by these techniques if this almost instantaneous ambiguity resolution becomes available.

Actual trends to integrate various systems based on identical or different sensor technologies (INS, GLONASS, etc) to overcome one or the other drawback of a specific single sensor systems is going to improve the performance of these positioning systems even further.

There is no doubt, that these positioning techniques with GPS are ready to enter into the field of detail surveying. It seems that we are just about to experience another revolution in surveying caused by positioning techniques with GPS.

Part V : Prediction of Ambiguity Resolution

Part V : Prediction of Ambiguity Resolution

Predicting the Ambiguity Resolution Performance :

Candidate Predictors

V : 1. Introduction

1.1 Objectives

Since the real-time processing capabilities for GPS positioning techniques using phase measurements are still limited, the demand for reliable and accurate algorithms to predict the amount of observations needed to fix the ambiguities successfully to integer values is obvious. Prediction algorithms have been proposed by several authors i.e. the "Differential GDOP" by Ron Hatch [HATCH, 1987] and the "Resolution of the Cycle Ambiguity" by Bertrand Merminod [MERMINOD, 1988] the RDOP's by C. Goad [GOAD, 1988] etc.

The case of rapid static positioning with the "Fast Ambiguity Resolution Approach" (FARA) without real - time processing capabilities demonstrates the need for an accurate and reliable ambiguity resolution predictor. One to five minutes worth of data suffice to resolve ambiguities using the "FARA" for distances up to 10 kilometers, as presented in Part III and IV. The question whether ambiguities can be fixed to integer values might depend on a single measurement. Therefore, such a predictor has to operate on the level of individual observations. It could be possible that the presence or absence of a single measurement or a small group of measurements to a specific satellite will decide whether or not the ambiguity resolution will be successful. It seems mandatory to use such prediction tools in real-time to inform the user when sufficient measurements have been taken to resolve ambiguities. The usage of such a predictor without a data link with the participating receiver has some severe limitations. One has to assume e.g. that every available satellite can actually be tracked by all the participating receivers and that no loss of lock occurs in the reference receiver.

To be able to use such a predictor on site requires in turn that it has to be simple and self-contained. The algorithms should work without any interference from the user and it should be tailored to the processing technique used for the final data processing.

This part of the document provides a survey of quality measures which could serve as a basis for a prediction approach. It will cover a few theoretical aspects dealing with measures of quality, namely precision and reliability as well as statistical testing procedures.

1.2 Requirements

The requirements for an ambiguity resolution prediction technique are summarized in the table below :

Table 1.2.1 : *Requirements for an Ambiguity Resolution Predictor*

(i)	It has to predict the ambiguity resolution performance on a very high level of confidence.
(ii)	The accuracy of the prediction has to be very high considering the short site occupation times needed to resolve ambiguities.
(iii)	The prediction algorithm must be used in real-time.
(iv)	It has to be tailored to the specific properties of the "FARA".
(v)	It should run in an automatic mode without any user interference.

V : 2. Quality Measures and Testing

There are two groups of quality measures, namely :

(i) The measures for **precision** given by the variance-covariance matrix for the observations (K_{ll}) and the variance-covariance matrix for the unknown parameters (K_{xx}) and

(ii) The measures for **reliability** given by the minimal detectable biases in the observations (internal reliability) or the minimal detectable biases in the unknown parameters (external reliability).

The indicators in group (i) are based on the information contained in the corresponding variance-covariance matrices. The indicators in group (ii) are derived using basic concepts from statistical hypothesis testing. A short introduction to the mathematical model used , to the possible measures for quality as well as the concepts of statistical hypothesis testing will follow. For a detailed description for these particular topics see (JUST, 1979), (VAN DER MAREL, 1989 and 1990), (VANICEK, KRAKIWSKY, 1986) and (PELZER, 1980 and 1985).

2.1 Functional and Stochastic Model

All measures of quality under consideration can be computed in an "a priori" phase, that means, without taking any measurements. The required information to compile these measures is contained in Table 2.1.1 . It can be computed as follows :

The functional model for the positioning task is based on the double-differenced phase observable. The unknown parameters are the coordinates of the survey marker (implicitly contained in the topocentric distances to the satellites) and the ambiguity parameters N_f^{nm}. The linearization of observation equations at time t (epoch k) assuming an approximate receiver position (x_0,y_0,z_0) yields the functional model in matrix representation as given by :

Table 2.1.1 : Information Basis for the Ambiguity Resolution Predictors

σ_0^2	:	the a priori variance of unit weight,
Q_{ll}	:	the matrix of cofactors for the observations,
A	:	the first design matrix and
Q_{xx}	:	the matrix of cofactors for the unknown parameters.

$$\boxed{\hat{l}_k = l_k + v_k = A_k\hat{x}} \tag{2.1}$$

where :

k	:	Epoch number,
$\hat{l}_k$	:	Vector of adjusted observations at epoch k,
l_k	:	Vector of observations at epoch k,
A_k	:	First design matrix at epoch k,
$\hat{x}$	:	Vector of unknown parameters and
v_k	:	Residuals.

The stochastic model is given by :

$$\boxed{K_{llk} = \sigma_0^2 \cdot Q_{llk} = \sigma_0^2 \cdot P_k^{-1}} \tag{2.2}$$

where :

K_{llk}	:	Variance-covariance matrix for the observations at epoch k,
σ_0^2	:	a priori variance of unit weight,
Q_{llk}	:	Matrix of cofactors for the

observations at epoch k and

P_k : Weight matrix for the observations at epoch k.

All mathematical correlations introduced in forming the double-differenced phase observables are rigorously treated, so the matrices in (2.2) above are fully populated.

Assuming that more than one observation epoch might be combined, the normal equation system after j observation epochs can be expressed by :

$$\left[\sum_{k=1}^{j} (A_k^T P_k A_k) \right] \cdot \hat{x}_j = \sum_{k=1}^{j} (A_k^T P_k l_k) \tag{2.3}$$

After j observation epochs the unknown parameters $\hat{x}_j$ are determined by :

$$\hat{x}_j = Q_{xxj} \cdot b_j \tag{2.4}$$

where :

$$Q_{xxj} = \left[\sum_{k=1}^{j} (A_k^T P_k A_k) \right]^{-1} = N_j^{-1}$$

$$b_j = \sum_{k=1}^{j} (A_k^T P_k l_k) \tag{2.5}$$

and :

N_j : Normal equation matrix after j epochs,

Q_{xxj} : Matrix of cofactors for the unknown parameters after j observation epochs.

Therefore, the cofactor matrix Q_{xxj} for the unknown parameters can be computed without any actual measurements. If the assumed the "a priori" variance σ_0^2 of unit weight is known, the variances and covariances for the unknown parameters can be computed by :

$$K_{xxj} = \sigma_0^2 \cdot Q_{xxj} \tag{2.6}$$

where :

K_{xxj}	:	Variance-covariance matrix for the unknown parameters and
σ_0^2	:	known "a priori" variance of the unit weight.

2.2 Measures for Precision

Six different measures for precision will be covered in the following. All these measures can be extracted directly from the variance-covariance matrix K_{xxj}.

2.2.1 The Variances before Ambiguity Resolution

The variances for individual parameters (coordinates and ambiguities) are given by :

$$\sigma_{xi}^2 = [K_{xxj}]_{ii} \ ; \ i = 1, \ldots, u \tag{2.7}$$

with :

σ_{xi}^2	:	Variance of unknown parameter i,
$[K_{xxj}]_{ii}$	:	i^{th} diagonal element of the variance-covariance matrix of the unknown parameters and
u	:	number of unknown parameters.

2.2.2 The Variances after Ambiguity Resolution

Assuming all ambiguities can be resolved to integer values, the remaining unknown parameters are the coordinates of the survey marker. The variances for these coordinate parameters are given by:

$$\sigma_{xi}^2 = [(N_{00j})^{-1}]_{ii} \; ; \; i = 1, \ldots, u \tag{2.8}$$

with :

σ_{xi}^2	:	Variance of unknown coordinate parameter i,
N_{00j}	:	Part of the matrix N_j which belongs to the coordinate parameters (x,y,z),
$[(N_{00j})^{-1}]_{ii}$	:	i^{th} diagonal element of the variance-covariance matrix for the coordinate parameters and
u	:	Number of unknown parameters (considering a single baseline u is set to 3) after ambiguity resolution.

2.2.3 The Trace of Q_{xxj} before Ambiguity Resolution

The trace of the Q_{xxj} matrix before ambiguity resolution is given by :

$$T_{xxj} = \left(\sum_{i=1}^{u} [Q_{xxj}]_{ii} \right)^{1/2} \tag{2.9}$$

with :

T_{xxj}	:	Square-root of the trace of Q_{xxj}.

In the navigation community the square-root of the trace of specific functional models are well known as the Dilution of Precision values (DOP's), e.g. the Geometrical Dilution of Precision (GDOP) and the Positional Dilution of Precision (PDOP) and so on. These DOP's are based on the functional model for positioning with code measurements in an undifferenced way. DOP's form indicators which consider not only one single unknown and the associated quality as with the variances in case 2.2.1 and 2.2.2 . They include all the unknown coordinate parameters and for certain DOP's also additional parameters i.e. clock parameters etc. That is one of the reasons why these DOP's are used to indicate the quality of a specific satellite constellation and hence the expected quality of the point positioning. Note that in our case equation (2.8) includes the variances for the coordinate parameters as well as the variances for the ambiguity parameters.

2.2.4 The Trace of Q_{xxj} after Ambiguity Resolution

The cofactor matrix of the unknown parameters after ambiguity resolution is given by $(N_{00j})^{-1}$. So the corresponding trace is given by :

$$\boxed{T_{xxj}^{*} = \left(\sum_{i=1}^{u} [(N_{00j})^{-1}]_{ii} \right)^{1/2}} \tag{2.10}$$

with :

T_{xxj}^{*}	:	Square-root of the trace of $(N_{00j})^{-1}$ and
$(N_{00j})^{-1}$	:	Cofactor matrix of the unknown parameters after ambiguity resolution.

In the case where u is set to three (three coordinate unknowns x,y,z) $T_{xxj}^{*} \cdot \sigma_0$ is well known as the so called "Helmert'scher (mittlerer) Punktfehler" (PELZER, 1985).

2.2.5 The Eigenvalues λ_i of K_{xxj} before Ambiguity Resolution

The eigenvalues of K_{xxj} can be determined by a so called spectral decomposition (for a detailed description see (PELZER, 1985)). K_{xxj} can be expressed by :

$$\boxed{K_{xxj} = S_{xxj} \cdot D_{xxj} \cdot S_{xxj}^{T}} \tag{2.11}$$

with :

S_{xxj}	:	the modal matrix containing the eigenvectors s_i and
D_{xxj}	:	the spectral matrix containing the eigenvalues λ_i as the diagonal elements.

These eigenvalues λ_i represent the length of the axes of a u-dimensional ellipsoid (u denotes the number of unknown parameters), whereas the eigenvectors themselves represent the direction of the axes. The shape and form of this u-dimensional hyperellipsoid can be used as an indicator for the integral quality of the point determination. The volume of the u-dimensional hyperellipsoid V_{xxj} is given by :

$$\boxed{V_{xxj} = \frac{2}{n} \pi^{n/2} \frac{1}{\Gamma(n/2)} \left(\prod_{i=1}^{u} \lambda_i \right)^{1/2}} \tag{2.12}$$

The proportionality factor k is dependent on the number of unknown parameters u. V_{xxj} is one of the commonly used quality criterion based on eigenvalues. The second criterion is the magnitude of the largest eigenvalue λ_{max}.

2.2.6 The Eigenvalues λ_i after Ambiguity Resolution

The eigenvalues for the unknown parameters after ambiguity resolution can be computed using the equations presented in 2.2.5 if K_{xxj} is replaced by $\sigma_0^2 * (N_{00j})^{-1}$. The number of unknown parameters is set to three (x,y,z - coordinates of the survey marker) considering the case of a single baseline.

2.3 Measures for Reliability

2.3.1 Internal and External Reliability

First we have to cover a few aspects of hypothesis testing, in particular the so called model test (JUST, 1979).

The mathematical model (functional and stochastic model) for an adjustment is based on a few assumptions when approximating the physical reality with a model. These assumptions are formulated either implicitly or explicitly in the mathematical model. The hypothesis that the selected mathematical model for the positioning adjustment is correct is usually called the null-hypothesis H_0. This null-hypothesis says that:

$$\mathbf{l} \sim N\,[\,\mathbf{Ax}, \sigma_0^2(\mathbf{Q_{ll}})]$$

(2.13)

the observations have a normal probability density function N with the expectation value **Ax** and the covariance matrix $\sigma_0^2(\mathbf{Q_{ll}})$
and

$$\hat{\mathbf{x}} \sim N\,[\,\mathbf{x}, \sigma_0^2(\mathbf{Q_{xx}})]$$

(2.14)

the estimated parameters have a normal probability density function with the expectation value **x** and the covariance matrix $\sigma_0^2(\mathbf{Q_{xx}})$.

Whether this null hypothesis is valid or not can be tested with the so called model test. The test statistic T_m is :

$$T_m = \frac{s_0^2}{\sigma_0^2} = \frac{\mathbf{v^T P v}}{\sigma_0^2(n-u)} = \frac{\mathbf{v^T P v}}{\sigma_0^2\, f}$$

(2.15)

where :

σ_0^2	:	a priori variance of unit weight,
s_0^2	:	a posteriori variance of unit weight,
n	:	Total number of observations,
u	:	Total number of unknown parameters and
f	:	Degree of freedom $f = n - u$.

Provided the null-hypothesis H_0 is correct, then T_m is distributed as:

$$H_0 : T_m \sim \chi^2\,[f,0]$$

(2.16)

The null-hypothesis is accepted, if

$$T_m < \chi^2_{1-\alpha}[f,0] \tag{2.17}$$

and rejected if

$$T_m \geq \chi^2_{1-\alpha}[f,0] \tag{2.18}$$

If the mathematical model had to be rejected, the reasons for this rejection need to be investigated. This is achieved by formulating so called alternative hypotheses which are based on the assumption that there are additional error sources or unmodelled systematic biases in the measurements. These alternative hypotheses H_{Ai} can be expressed by:

$$H_{Ai} : \mathbf{l} \sim N[\,\mathbf{Ax} + \mathbf{C}_i\nabla_i,\ \sigma_0^2\mathbf{Q}_{ll}] \tag{2.19}$$

$C_i\nabla_i$ defines a possible model error in the observations. ∇_i denotes the vector of model errors and C_i specifies the possible errors. The definition of alternatives to the null-hypothesis is crucial and usually not a trivial task. The model errors ∇_i can be computed as well as the corresponding cofactor matrix $Q_{\nabla\nabla}$. Again a test statistic can be employed to test whether the null-hypothesis H_0 or the alternative H_{Ai} should be accepted. The test statistic is:

$$T_{Ai} = \nabla_i^T Q_{\nabla\nabla} \nabla_i = \lambda_0 \tag{2.20}$$

where λ_0 denotes the so called non-centrality parameter.

Provided the null-hypothesis is correct then the test statistic is distributed as:

$$H_0 : T_{Ai} \sim \chi^2[f,0] \tag{2.21}$$

If the alternative hypothesis is correct then the test statistic is distributed as:

$$H_{Ai} : T_{Ai} \sim \chi^2[f,\lambda_i] \tag{2.22}$$

The null-hypothesis is rejected only if:

$$T_{Ai} = \nabla_i^T Q_{\nabla\nabla} \nabla_i \geq \chi^2_{1-\alpha}[f,0] \tag{2.23}$$

All possible and feasible alternatives have to be evaluated in terms of the resulting test statistic T_{Ai}. If one or more alternatives can be accepted, appropriate actions have to be taken to amend the adjustment. In the case where more than one alternative can be accepted, only the most likely one should be treated at a time, because the alternatives are correlated. The remaining alternatives will have to be tested again as soon as the recomputations have been carried out. For a detailed introduction to this model test see (JUST,1979) or (VAN DER MAREL, 1989, 1990).

Let us consider again the aspects of reliability. Actually, the very same procedure described above but in reverse order is used to determine the measures of reliability. Fixing the non-centrality parameter λ_0 for all alternatives based on the specification of a level of significance as well as the power of the test, the so called minimal detectable biases can be determined. The **internal reliability** specifies then the minimal biases in the observations which still can be detected by means of hypothesis testing. Biases in the observation given by $C_i\nabla_i = \nabla_{il}$ can be detected if and only if :

$$\nabla_{il}^T (Q_{ll})^{-1} \nabla_{il} \geq \lambda_0 \tag{2.24}$$

where :

∇_{il}	:	minimal detectable biases in the observations **l** ,
$(Q_{ll})^{-1}$	:	weight matrix for the observations **l** and
λ_0	:	non-centrality parameter.

The effects of these minimal detectable biases in the observations ∇_{il} on the final result is called **external reliability**. It can be computed by :

$$\nabla_{ix} = (Q_{xx}) A^T (Q_{ll})^{-1} \nabla_{il} \tag{2.25}$$

∇_{ix} denotes the minimal detectable biases in the estimated parameters. The biases ∇_{ix} in the estimated parameters can be detected if and only if :

$$\boxed{\nabla_{ix}^T (Q_{xx})^{-1} \nabla_{ix} \geq \lambda_0} \qquad (2.26)$$

The question is how can these measures of precision and reliability be exploited to predict whether a successful ambiguity resolution for a certain measuring scenario will be possible or not. Before this question is addressed we have to treat the effect of resolving ambiguities on the estimated parameters.

2.3.2 Effects of Resolving Ambiguities on the Remaining Estimated Parameters

The transition from real-valued ambiguities to integer ambiguities from the adjustment point of view consists of the following steps :

The solution vector x_j after an initial adjustment (without ambiguity resolution) can be grouped in sub-vectors of the form :

$$\boxed{\hat{x}_j^T = [\ \hat{x}_{jC}^T, \hat{x}_{jN1}^T, \ \hat{x}_{jN2}^T, ..., \hat{x}_{jNn}^T\ ]} \qquad (2.27)$$

where :

$\hat{x}_{jC}^T$: denotes the estimated coordinates after the initial adjustment and

$\hat{x}_{jNi}^T$: denotes the estimates of the real-valued ambiguities for the ambiguities in group i.

A group of ambiguity parameters can consist of L_1 or L_2 ambiguity parameters for a specific measuring period. This measuring period is characterized by continuous phase measurements. A new measuring period, usually referred to as a session, calls for a new set of ambiguity parameters either for L_1 or L_2 ambiguities. The corresponding structure of the normal equation system **N** before ambiguity resolution is :

$$\begin{vmatrix} N_{j00} & N_{j01} & N_{j02} & \dots & N_{j0g} \\ N_{j10} & N_{j11} & 0 & \dots & 0 \\ N_{j20} & 0 & N_{j22} & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ N_{jg0} & 0 & 0 & \dots & N_{jgg} \end{vmatrix} \begin{vmatrix} x_{jC} \\ x_{jN1} \\ x_{jN2} \\ \dots \\ x_{jNg} \end{vmatrix} = \begin{vmatrix} b_{jC} \\ b_{jN1} \\ b_{jN2} \\ \dots \\ b_{jNg} \end{vmatrix}$$

(2.28)

where :

N_{j00}	:	contains the elements of N_j for the coordinate unknowns,
N_{jii}	:	contains the elements of N_j for the i^{th} group of ambiguity parameters,
N_{j0i}, N_{ji0}	:	contain the elements of N_j expressing the correlations between ambiguity parameters and coordinates and
g	:	denotes the number of ambiguity groups.

Note that there is no mathematical correlation between different groups of ambiguities. There is only one group of coordinate parameters. In the case of single baseline determinations there are three coordinate parameters to be estimated.

Assuming that the resolution of the real-valued estimates to integers is possible each individual real-valued ambiguity has got its corresponding integer ambiguity contained in x_{jNi}^* . The new position vector x_{jC}^* as a result of fixing the ambiguity parameters to integer values can be computed by :

$$\hat{x}_{jC}^* = (N_j^*)^{-1} \cdot b_j^*$$

(2.29)

where :

x_{jC}^*	:	denotes the vector of estimated coordinates after ambiguity resolution,
N_j^*	:	denotes the normal equation system after j epochs (no more ambiguity parameters included) and
b_j^*	:	denotes a vector containing the sum of (shortened) observation vectors transformed into the solution space.

N_j^* and b_j^* are given by the following equations :

$$\boxed{N_j^* = N_{j00}}$$

(2.30)

and

$$\boxed{b_j^* = b_j - \sum_{i=1}^{g} N_{j0i} \cdot x_{jNi}^*}$$

(2.31)

where :

x_{jNi}^* : denote the integer ambiguities for the i^{th} group and

g : denotes the number of ambiguity groups.

The solution vector after ambiguity resolution is given by :

$$\boxed{\hat{x}_{jC}^* = (N_{j00})^{-1} \cdot \left[b_j - \sum_{i=1}^{g} N_{j0i} \cdot x_{jNi}^* \right]}$$

(2.32)

2.3.3 Formulation of alternatives

Under the assumption, that the null hypothesis is still valid after ambiguity resolution the estimated parameters are distributed as

$$\boxed{\hat{x}_{jC}^* \sim N[x_{jC}^*, \sigma_0^2(Q_{xx})]}$$

(2.33)

The model test (2.13) can be employed to test, whether there are errors in the mathematical model or not. Following the concepts of reliability one may ask, what are the minimal detectable biases in the estimated parameters for a particular measuring scenario? In order to be able to determine these minimal detectable biases ∇_{ix}, appropriate alternatives H_{Ai} have to be formulated. The selected error model reads :

$$\boxed{x_{jNf}^{*} = x_{jNi}^{*} + \Delta N_i}$$

(2.34)

where :

ΔN_i : denotes the vector of errors for the ambiguity parameters.

The effect of the errors specified by (2.34) on the estimated parameters can be computed by :

$$\boxed{\nabla_{ix} = (N_{j00}^{-1} \cdot N_{j0i} \cdot \Delta N_i)}$$

(2.35)

where :

∇_{ix} : denotes the biases in the estimated parameters introduced by changing the integer ambiguity vector x_{jNi}^{*}.

Therefore the alternative H_{Ai} reads :

$$\boxed{\hat{x}_{jC}^{*} \sim N[x_{jC}^{*} + \nabla_{ix}, \sigma_0^2(Q_{xx})]}$$

(2.36)

The error ΔN_i specified by (2.34) can be detected by means of hypothesis testing if and only if :

$$\boxed{\nabla_{ix}^{T}(Q_{xx})^{-1}\nabla_{ix} \geq \lambda_0}$$

(2.37)

The non-centrality parameter λ_0 is given. It is usually determined by selecting the power of the test and the level of significance. Replacing ∇_{ix} in equation (2.37) by (2.35) yields :

$$\boxed{\lambda_{ix} = \Delta N_i^{T} \cdot N_{j0i}^{T} \cdot Q_{xx} \cdot N_{j0i} \cdot \Delta N_i \geq \lambda_0}$$

(2.38)

Equation (2.38) represent the working formula to test whether a particular hypothesized bias in the integer valued ambiguities given by ΔN_i can be detected or not.

2.3.4 Predicting the Ability to Resolve Ambiguities

Equation (2.38) could be employed as a prediction tool. Ambiguity resolution strategies based on general search procedures rely on the assumption that the biases introduced by fixing ambiguities wrongly can be detected by means of hypothesis testing. The correct solution will yield usually a minimum a posteriori variance of unit weight and all the biased solutions will be sufficiently different from the correct solution to guarantee a reliable selection of the final set of integers (see Part III). From experience we know, that this is only true if the observed satellite geometry and the measurement data meet some minimum requirements. Utilizing the procedures derived above should enable us to evaluate whether a particular satellite constellation along with the corresponding measurements has the required sensitivity and redundancy to detect biases caused by typical errors in the ambiguity resolution. Such typical errors have to be modelled in form of alternatives (H_{Ai}) to the null-hypothesis (H_0). If an evaluation of all the alternatives yields that all the formulated errors in these alternatives should be detectable by a model test, it can be assumed, that this particular satellite constellation with the corresponding measurement scenario will enable to resolve ambiguities successfully.

Tests with actual measurements taken under different satellite constellations and various measuring scenarios will have to prove the validity of such an ambiguity prediction approach.

V : 3. Remarks Concerning an Instantaneous Ambiguity Resolution

Let us assume that we observe a special satellite configuration at j subsequent epochs t_i :

$$t_i = t_1 + (1 - i) \cdot \Delta t, \; i = 1, 2, ..., j \tag{3.1}$$

where :

t_i	:	Observation times and
Δt	:	Sampling time interval.

What is the behavior of the formal accuracies for the unknown parameters if $\Delta t \rightarrow 0$? How does this process affect the ambiguity resolution performance ?

To find an answer we start from a simplified double difference observation equation (see also Part III equation (3.1.1)) :

$$dd(l)_i + v_i = dd(R)_i + \lambda \cdot N, i = 1, 2, ..., j \tag{3.2}$$

In the ambiguity resolution process we first have to solve for the coordinates of the receiver 2 with respect to receiver 1 (corresponding information contained in $dd(R)_i$) and for N (real valued estimation of ambiguities). In principle it is possible to split up this problem by first analyzing instead of (3.2) the following observation equation :

$$dd(l)_i + v_i - dd(l)_1 + v_1 = dd(R)_i - dd(R)_1, \; i = 2, 3, ..., j \tag{3.3}$$

The resulting coordinates and the formal accuracies by analyzing (3.2) or (3.3) will be almost identical. We now develop $dd(R)_i$ on the right hand side of (3.3) into a Taylor series around t_1 and obtain (series truncation after first order terms) :

$$\boxed{dd(R)_i - dd(R)_1 = (i-1) \bullet (dd(R)_1)^{(1)} \bullet \Delta t, \; i = 2, 3, \ldots, j} \tag{3.4}$$

This result can now be introduced into equation (3.3) to give approximate observation equations :

$$\boxed{(i-1) \bullet (dd(R)_1)^{(1)} \bullet \Delta t = dd(l)_i - dd(l)_1 + v_{i1}, \; i = 2, 3, \ldots, j} \tag{3.5}$$

In order to actually use the observation equation (3.5) we would have to linearize $(dd(R)_1)^{(1)}$ as a function of the coordinates. However, for our purpose this is not necessary, because we already see what will happen with the resulting normal equation matrix **N**. Due to the common factor Δt in all the observation equations (3.5) we may write :

$$\boxed{\begin{array}{ll} N \sim \Delta t^2 & , \quad \Delta t \rightarrow 0 \\ Q = N^{-1} \sim \Delta t^{-2} & , \quad \Delta t \rightarrow 0 \end{array}} \tag{3.6}$$

Since the diagonal elements of **Q** are proportional to the variances of the estimated coordinates, we may also conclude, that the formal accuracies of the estimated coordinates behave like :

$$\boxed{\sigma(x_i) \sim \Delta t^{-1} \quad , \quad \Delta t \rightarrow 0} \tag{3.7}$$

The resulting coordinates (contained in $dd(R)_i$) can now be introduced into equation (3.2) in order to estimate the ambiguities (real valued estimates) :

$$\boxed{dd(l)_i - dd(R)_i + v_i = \lambda \bullet N, \; i = 1, 2, \ldots, j} \tag{3.8}$$

We see from equation (3.8) that the formal accuracies for the ambiguity estimates will be almost completely determined by the accuracies of $dd(R)_i$, and therefore, by the formal accuracies of the coordinates estimated with equations (3.5). In analogy to (3.7) we will have :

$$\boxed{\sigma(N) \sim \Delta t^{-1} \quad , \quad \Delta t \rightarrow 0} \tag{3.9}$$

This means that the individual search ranges for the ambiguity resolution process grow as $\sim \Delta t^{-1}$ for $\Delta t \rightarrow 0$.

It is remarkable that the same is not true, if we look at the linear combination :

$$N1 - \frac{\lambda_2}{\lambda_1} \cdot N2 \tag{3.10}$$

when analyzing dual frequency data.

By forming the differences between two equations of type (3.2) (same pair of satellites, same epoch i, different carriers) it is easy to verify that

$$\sigma(N1 - \frac{\lambda_2}{\lambda_1} \cdot N2) \sim \Delta t^0 \tag{3.11}$$

which means that we can establish the relationship between L_1 - and L_2 - ambiguities with the same high accuracy independent on the length of the site occupation time.

Let us visualize these two different measuring scenarios (the L1 only and the L1 + L2 case) in order to show the differences in the process of ambiguity resolution considering the reduction of site occupation time.

Figure 3.1 shows the impact of doubling the sampling time interval $\Delta t \rightarrow 2 \cdot \Delta t$ on the ambiguity resolution process for single band observations. According to (3.9) the search ranges for individual ambiguities are reduced by a factor of 2 when Δt is replaced by $2 \cdot \Delta t$. Therefore, the total number of integer valued ambiguity combinations to be analyzed in the resolution process (all grid points within the square of Figure 3.1) grows inverse proportional with the change in the sampling time interval raised to power of the number of ambiguities.

Considering the case where L_1 and L_2 measurements are available, the situation changes because of (3.10) (see also Part III, 3.5). The ellipses in Figure 3.2 show the two dimensional search ranges for Δt and $2 \cdot \Delta t$. In contrary to the L1 case above not both of the ellipse-axes grow when reducing the observation time interval. The reasons can be seen in the fact that the differences of simultaneous L_1 and L_2 measurements are free of geometrical relationships. The magnitude of the small axis is determined by the accuracy of individual phase measurements (L_1 or L_2) which is

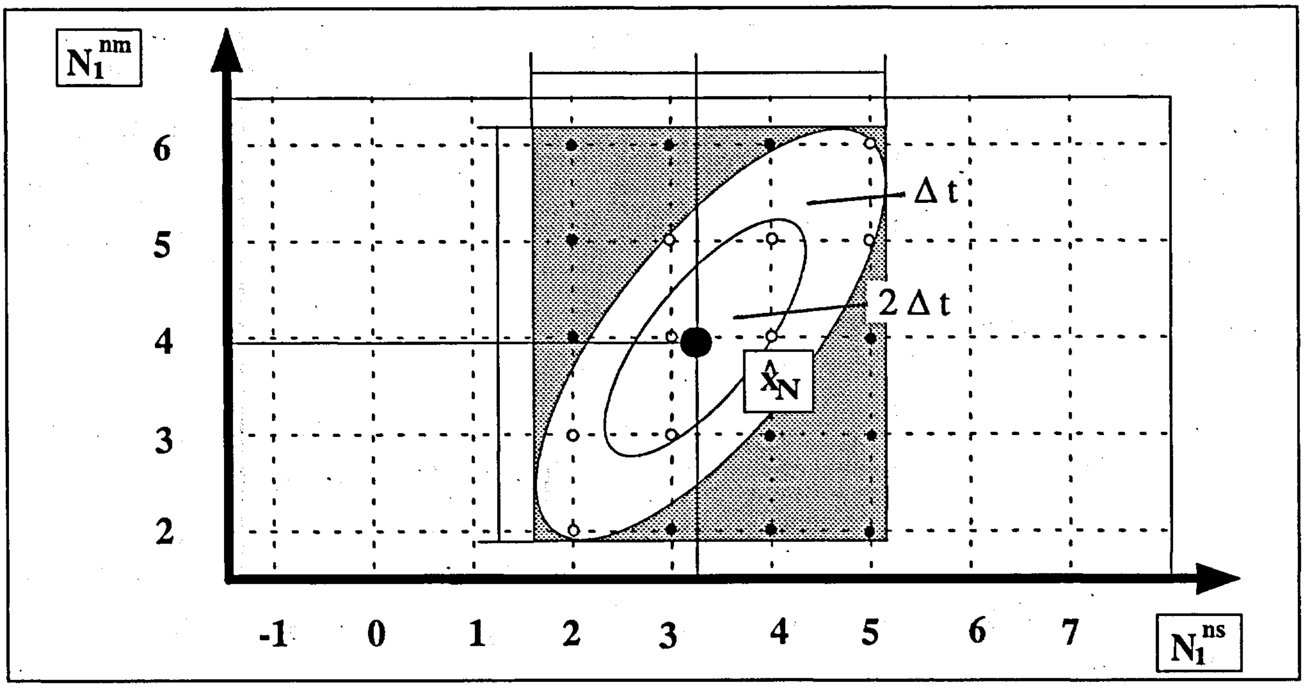


Figure 3.1 : Impact of doubling the sampling time interval on the ambiguity resolution process for the L_1 only case.

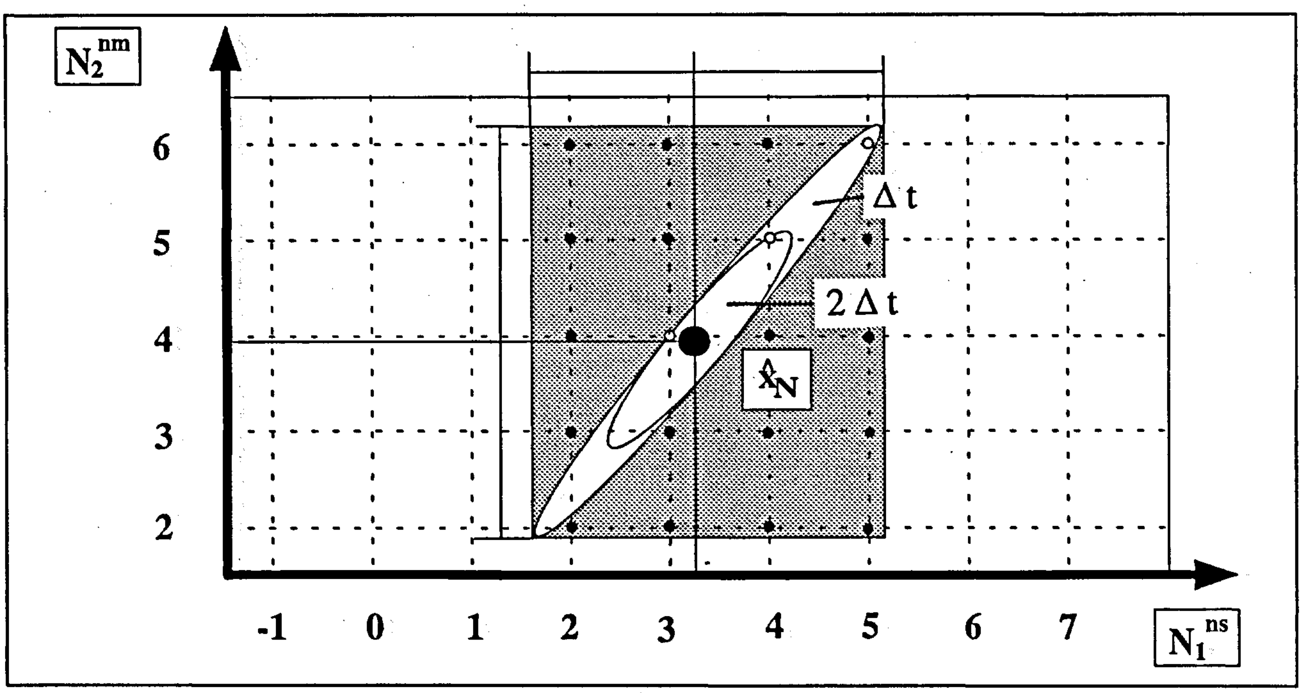


Figure 3.2 : Impact of doubling the sampling time interval on the ambiguity resolution process for the $L_1 + L_2$ case.

usually high and not dependent on the site occupation time. Therefore, the number of ambiguity combinations to be analyzed in the L_1 and L_2 resolution process does not grow according to the same rules as for the L1 only case. Actual results presented in Part IV (4.8.3) show that the ambiguity resolution with 15 seconds L_1 and L_2 data is successful. This might not yet represent the shortest site occupation time which allows a successful ambiguity resolution.

In summary we can draw the following conclusions :

(i) An instantaneous ambiguity resolution with just one epoch worth of measurements is impossible. A minimal change in the positions of satellites is required.

(ii) The formal accuracies for coordinates and ambiguities grow inverse proportional to the reduction of site occupation time (Δt^{-1}). Thus, the number of integer ambiguity combinations to be analyzed with a general search technique grows proportional to $(\Delta t^{-1})^n$ where n is the number of ambiguities.

(iii) The achievable ambiguity resolution performance with L1 data is limited by the rapidly growing number of different ambiguity combinations to be analyzed. Results indicate that the limit with a six satellite constellation is in the order of two to three minutes site occupation time.

(iv) The performance of ambiguity resolution with L_1 and L_2 data is much better due to the geometry free linear combinations between simultaneous L_1 and L_2 measurements. Successful ambiguity resolution with 15 seconds site occupation time has been demonstrated. The actual limitations are not yet clear.

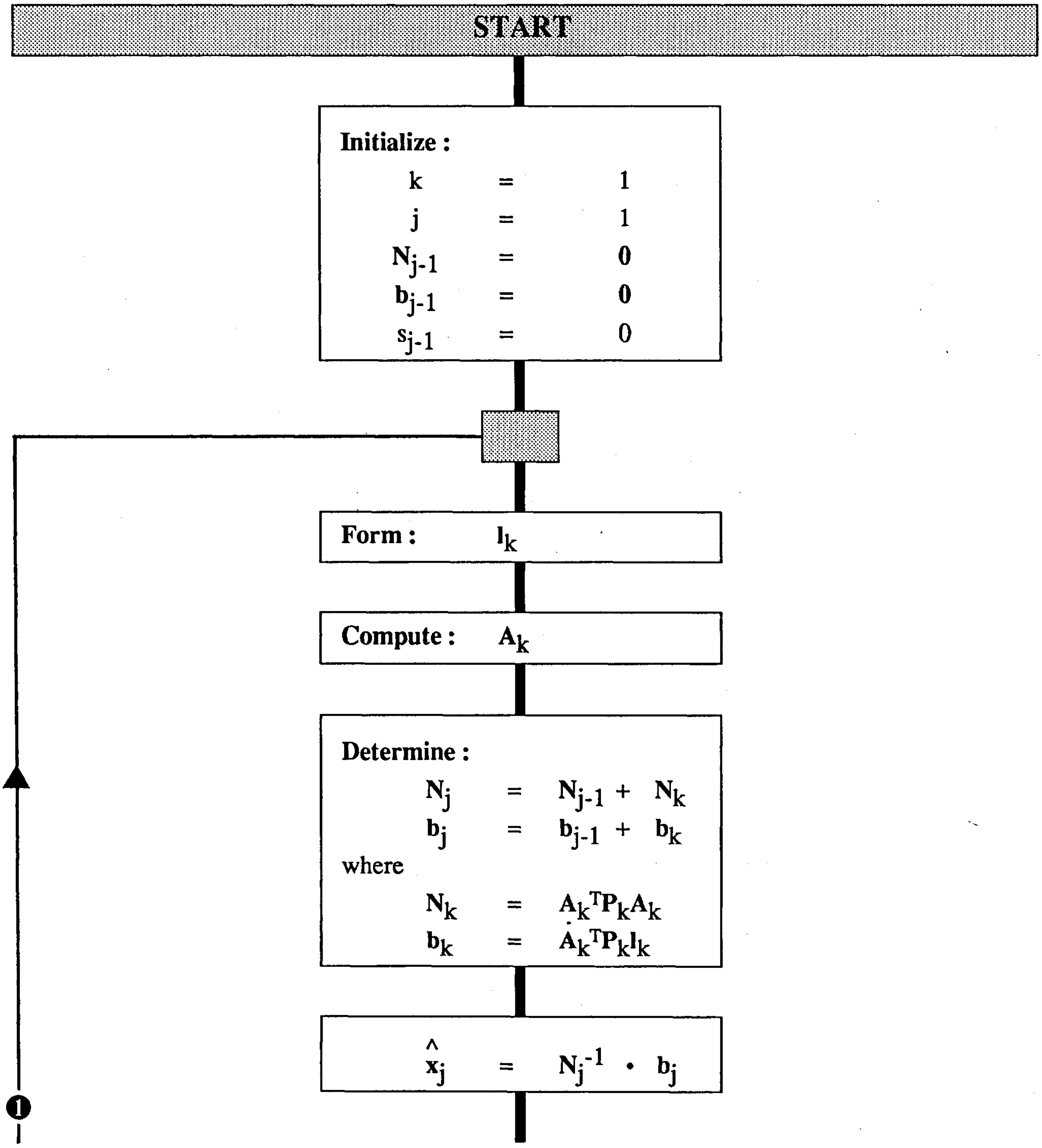
Appendix A : "FARA" Flow Diagram

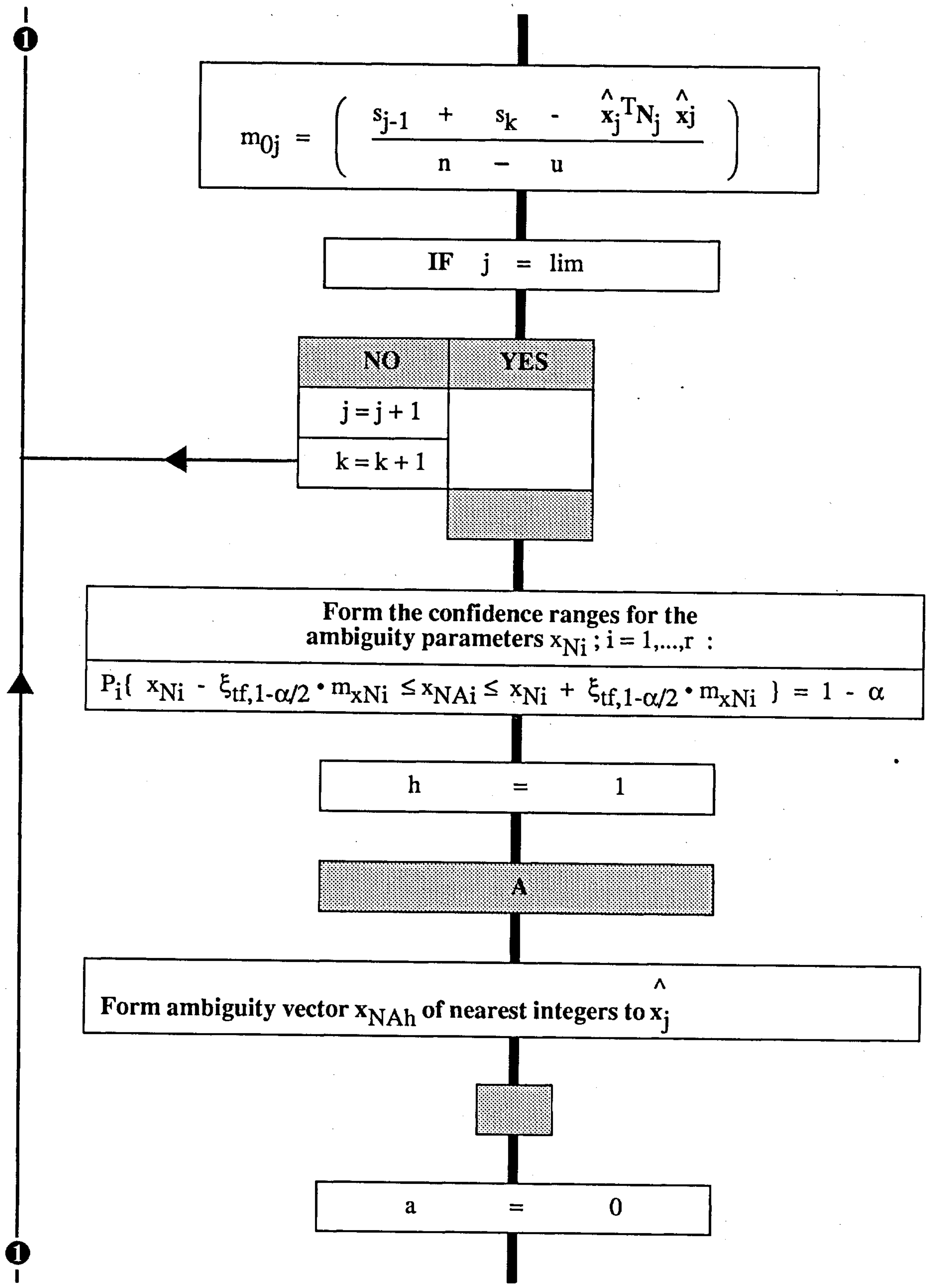
Apppendix A : "FARA" Flow Diagram

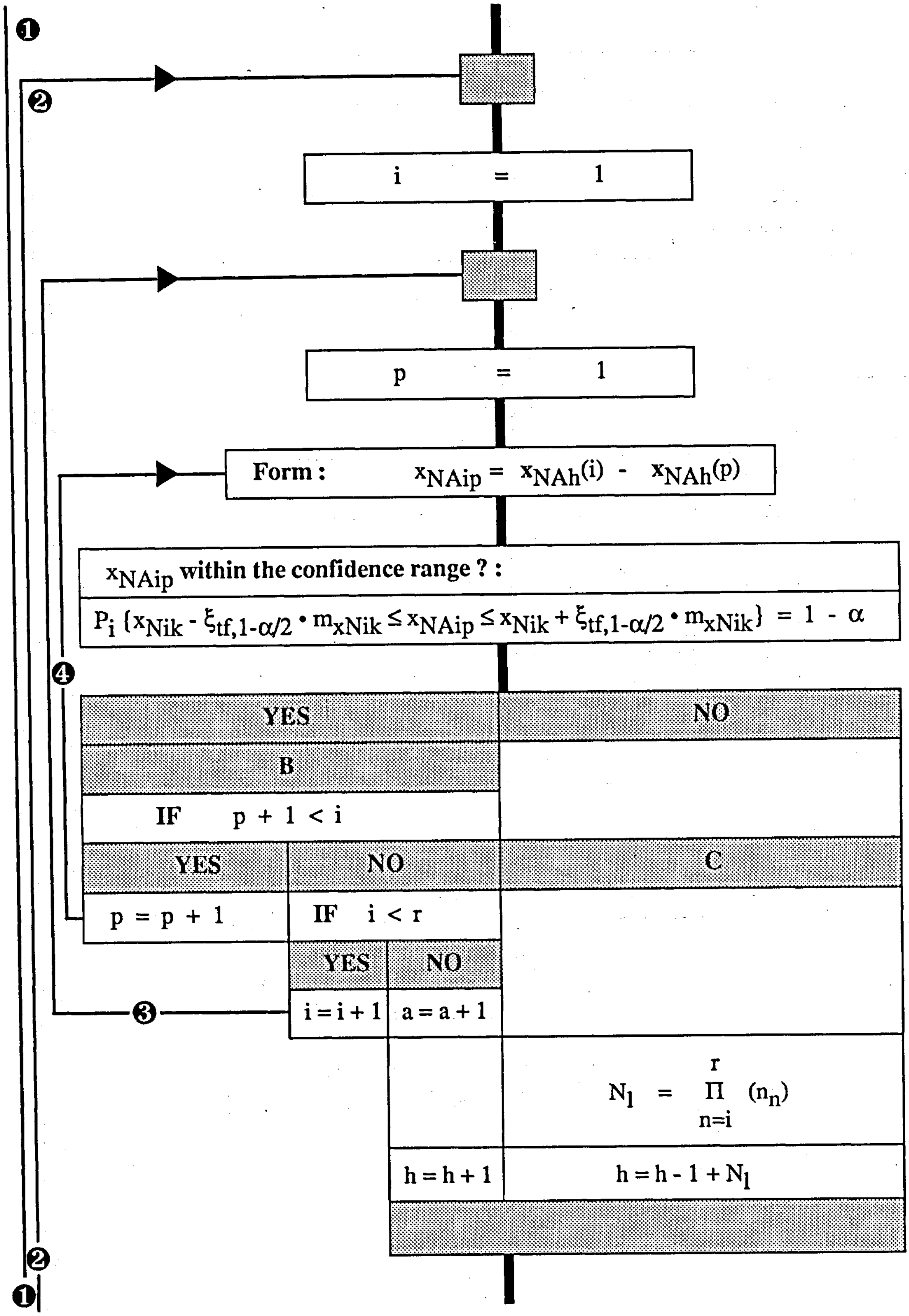
The Fast Ambiguity Resolution Approach summarized in a Flow Diagramm

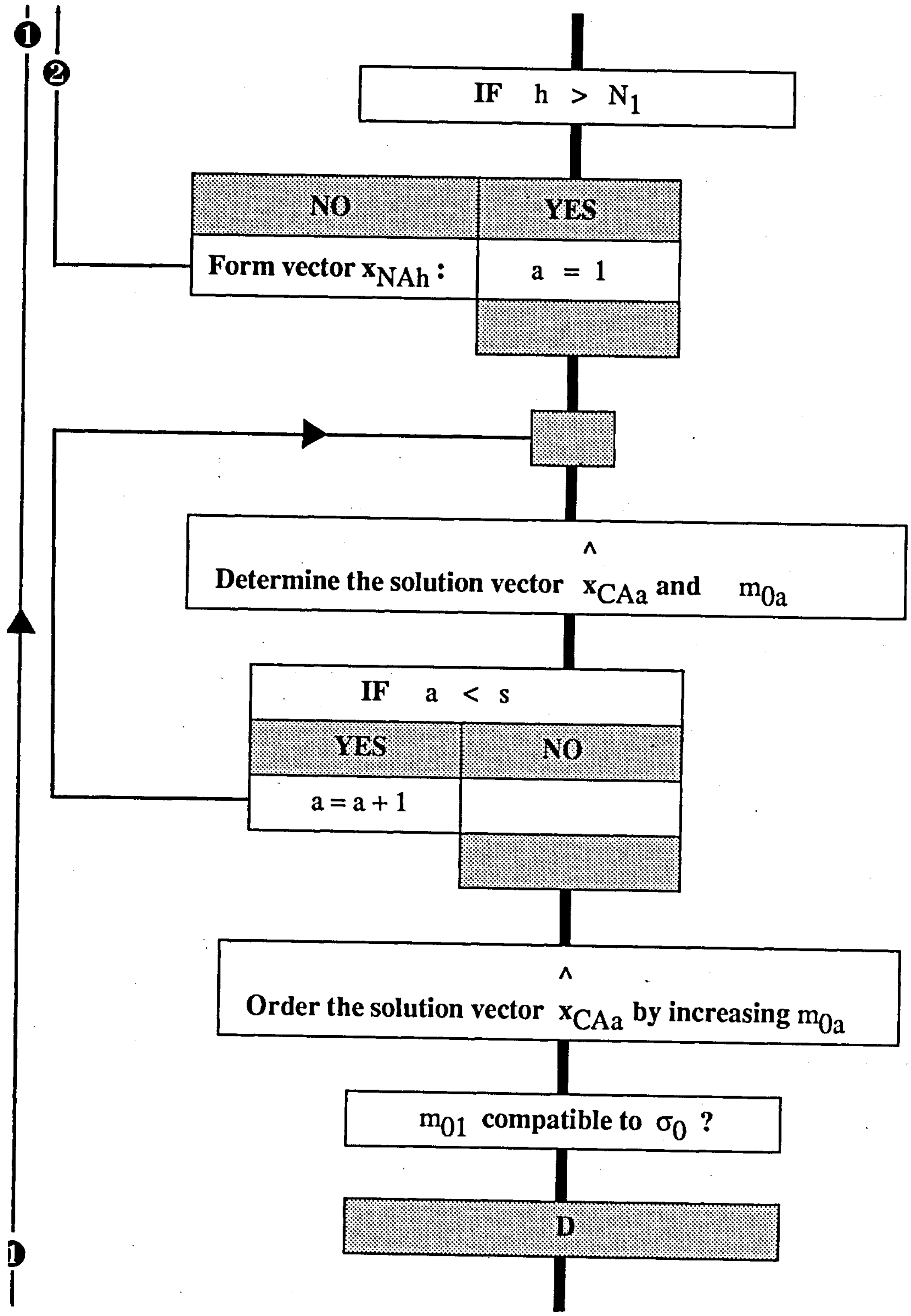
1. The "FARA" Flow Diagram

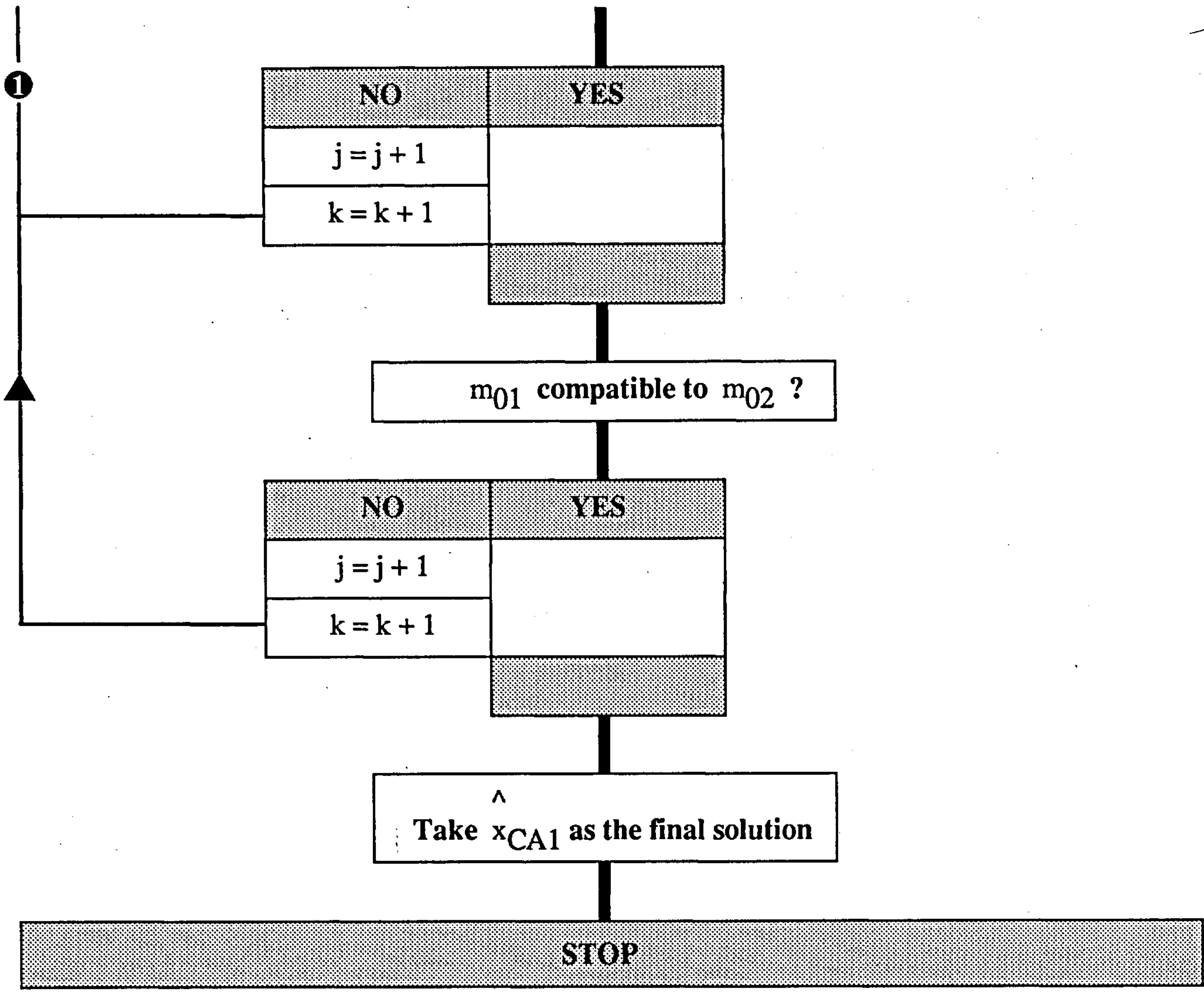
1.1 The General Flow



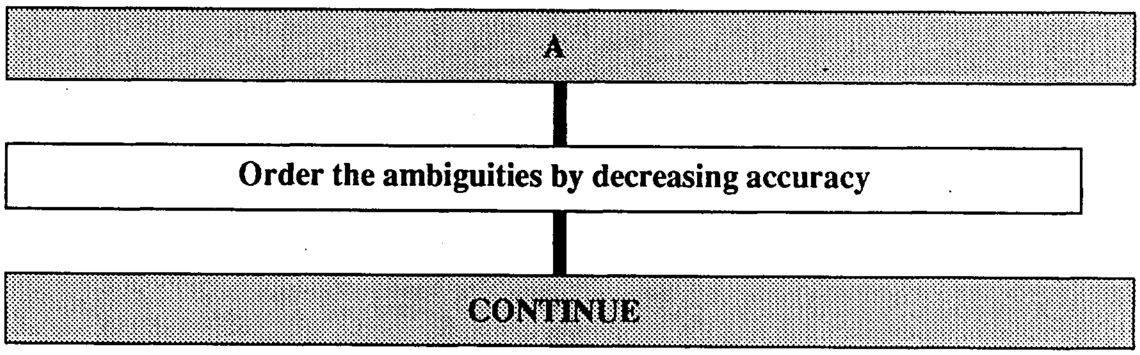

$m_{0j} = \left(\frac{s_{j-1} + s_k - \hat{x}_j^T N_j \hat{x}_j}{n - u} \right)$
IF j = lim
NO
YES
j = j + 1
k = k + 1
Form the confidence ranges for the ambiguity parameters x_{Ni} ; i = 1,...,r :
$P_i\{ x_{Ni} - \xi_{tf,1-\alpha/2} \cdot m_{xNi} \leq x_{NAi} \leq x_{Ni} + \xi_{tf,1-\alpha/2} \cdot m_{xNi} \} = 1 - \alpha$
h = 1
A
Form ambiguity vector x_{NAh} of nearest integers to $\hat{x}_j$
a = 0


1
2
$i = 1$
$p = 1$
Form : $x_{NAip} = x_{NAh}(i) - x_{NAh}(p)$
x_{NAip} within the confidence range ? :
$P_i \{x_{Nik} - \xi_{tf,1-\alpha/2} \cdot m_{xNik} \leq x_{NAip} \leq x_{Nik} + \xi_{tf,1-\alpha/2} \cdot m_{xNik}\} = 1 - \alpha$
4
YES
NO
B
IF $p + 1 < i$
YES
NO
C
$p = p + 1$
IF $i < r$
YES
NO
3
$i = i + 1$
$a = a + 1$
$N_l = \prod_{n=i}^{r} (n_n)$
$h = h + 1$
$h = h - 1 + N_l$
2
1


IF h > N_1
NO
YES
Form vector x_{NAh} :
a = 1
Determine the solution vector $\hat{x}_{CAa}$ and m_{0a}
IF a < s
YES
NO
a = a + 1
Order the solution vector $\hat{x}_{CAa}$ by increasing m_{0a}
m_{01} compatible to σ_0 ?
D


❶
NO
YES
j = j + 1
k = k + 1
m01 compatible to m02 ?
NO
YES
j = j + 1
k = k + 1
Take x̂CA1 as the final solution
STOP

1.2 Option A :



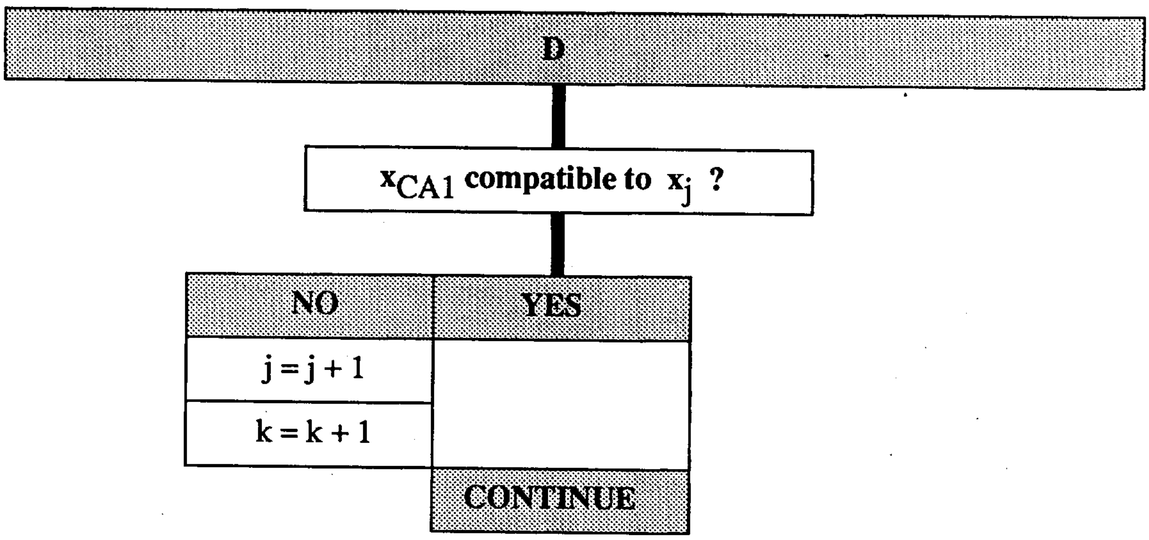
1.3 Option B :

B

Are there L1 and L2 measurement ? :

NO	**YES**	
	x_{NAip} **within the confidence range ? :**	
	$P_i \{ x_{Lik} - \xi_{tf,1-\alpha/2} \cdot m_{xLik} \leq x_{LAip} \leq x_{Lik} + \xi_{tf,1-\alpha/2} \cdot m_{xLik} \} = 1 - \alpha$	
	YES	**NO**
CONTINUE		**GOTO C**

1.4 Option D :



2. Terms and Definitions

k	:=	current measurement epoch
j	:=	current measurement epoch number
N_{j-1}	:=	accumulated normal equation system at epoch j - 1
b_{j-1}	:=	accumulated vectors $A_k^T P_k l_k$ after j - 1 epoches
s_{j-1}	:=	denotes $m_0^2 \cdot (n - u)$ at epoch j - 1
l_k	:=	vector of shortened observations in the sense observed minus computed
A_k	:=	First design matrix at epoch k
N_j	:=	Normal equation system after j observation epochs
N_k	:=	Normal equation system at epoch k
b_j	:=	sum of vectors $A_k^T P_k l_k$ after j epochs
b_k	:=	vector $A_k^T P_k l_k$ at epoch k
P_k	:=	weight matrix for the observation at epoch k
N_k	:=	normal equation system at epoch k ; $A_k^T P_k A_k$
$\hat{x}_j$	:=	solution vector after j epochs
N_j^{-1}	:=	inverse normal equation system after j epochs or the cofactor matrix for the estimated parameters
s_k	:=	$l_k^T P_k l_k$ at epoch k
m_{0j}	:=	rms a posteriori after j observation epochs
n	:=	number of measurements after j epochs
u	:=	number of unknown parameters
lim	:=	user defined parameter which determines the number of observation epochs to be summed up before an attempt to resolve ambiguities is undertaken
x_{Ni}	:=	ambiguity parameter i out of the solution vector x_j
$P_i\{...\}$	:=	probability statement
$\xi_{tf,1-\alpha/2}$	:=	upper and lower range width of the two-tailed confidence

		range 1 - α based on Student's probability density function t with f degrees of freedom
m_{xNi}	:=	rms a posteriori for the ambiguity parameter i out of the solution vector x_j
x_{NAi}	:=	integer-valued alternative for the ambiguity parameter x_{Ni}
1 - α	:=	confidence level
α	:=	error probability of the first kind (significance level)
h	:=	number of the alternative ambiguity vector x_{NA}
x_{NAh}	:=	alternative integer-valued ambiguity vector h
a	:=	current number of accepted alternative integer-valued ambiguity vectors x_{NAh}
i, p	:=	running variables for the loop
x_{NAip}	:=	integer-valued difference of two alternatives x_{NAi} and x_{NAp}
m_{xNip}	:=	a posteriori rms error for the difference of two alternatives x_{NAi} and x_{NAp}
r	:=	total number of ambiguity parameters (L_1 and L_2)
n_n	:=	number of integer values in the confidence range
N_1	:=	maximum number of alternative vectors to be tested
x_{CAa}	:=	solution vector which corresponds to the integer alternative ambiguity vector x_{NAa}
m_{0a}	:=	a posteriori rms error
s	:=	total number of accepted alternative vectors
m_{01}	:=	smallest a posteriori rms error
σ_0	:=	a priori rms error
x_{Lik}	:=	difference of an L_1 and an L_2 measurement to the same satellite
m_{xLik}	:=	rms error of the difference of an L_1 and an L_2 measurement to the identical satellite pair

Appendix B : References

Appendix B : References

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and

Text Material

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